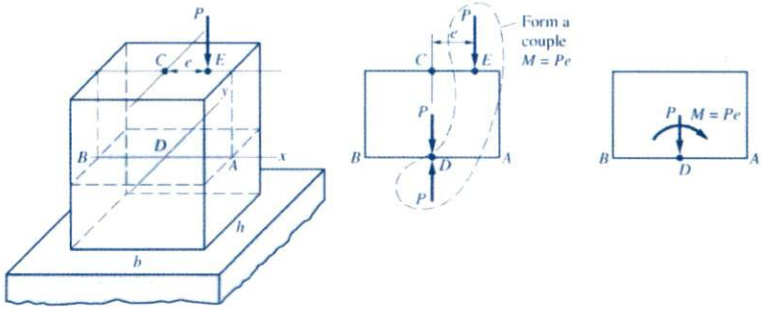
Eccentrically Loaded Members

- Another case of combined stress due to axial and bending effects arises when an axial load acts away from the centroidal axis of a member. Such a load is, therefore, referred to as an eccentric load.
- Eccentric loads, by their nature, subject the member to axial stress and to a bending stress. The bending stress develops from the moment caused by the load, multiplied by the eccentric distance or eccentricity.
- The method of superposition can be applied to a short compression member that has small deflections and will not buckle under compressive loads.



An eccentric axial load can be replaced by a concentric force and a couple.

The original downward force at E and the upward force at D form a couple M = Pe. The system is thus reduced to a concentric force P at the centroid D and a couple M = Pe, as shown above.

The combined stresses created by the axial force and bending moment can be determined in the same way as in section 18-2.

The axial force produces a uniform compressive stress throughout the section.
The bending moment produces maximum compressive stress at A and maximum tensile stress at B.

By superposition, the normal stresses at A and B are:

$$\sigma_A = \frac{-P}{A} - \frac{M}{S} \quad (18\text{-}1)$$ (Normal stress at A is always compressive)

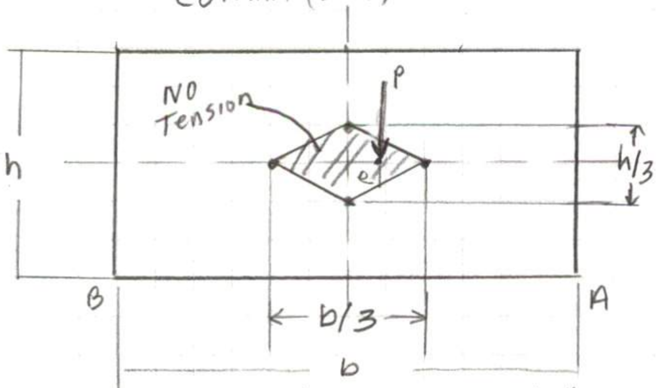
$$\sigma_B = \frac{-P}{A} + \frac{M}{S} \quad (18\text{-}2)$$ (Normal stress at B may be compressive, tensile, or zero, depends on e)

Maximum Eccentricty

For some materials that are weak in tension, keeping the eccentricty of a compressive load to a certain maximum limit so that no tensile stress develops anywhere in the member may be important.

Limit of Eccentricty for Solid Rectangular Section

Column (b×h)



"Middle Third Rule"

P on Right, same for P on left

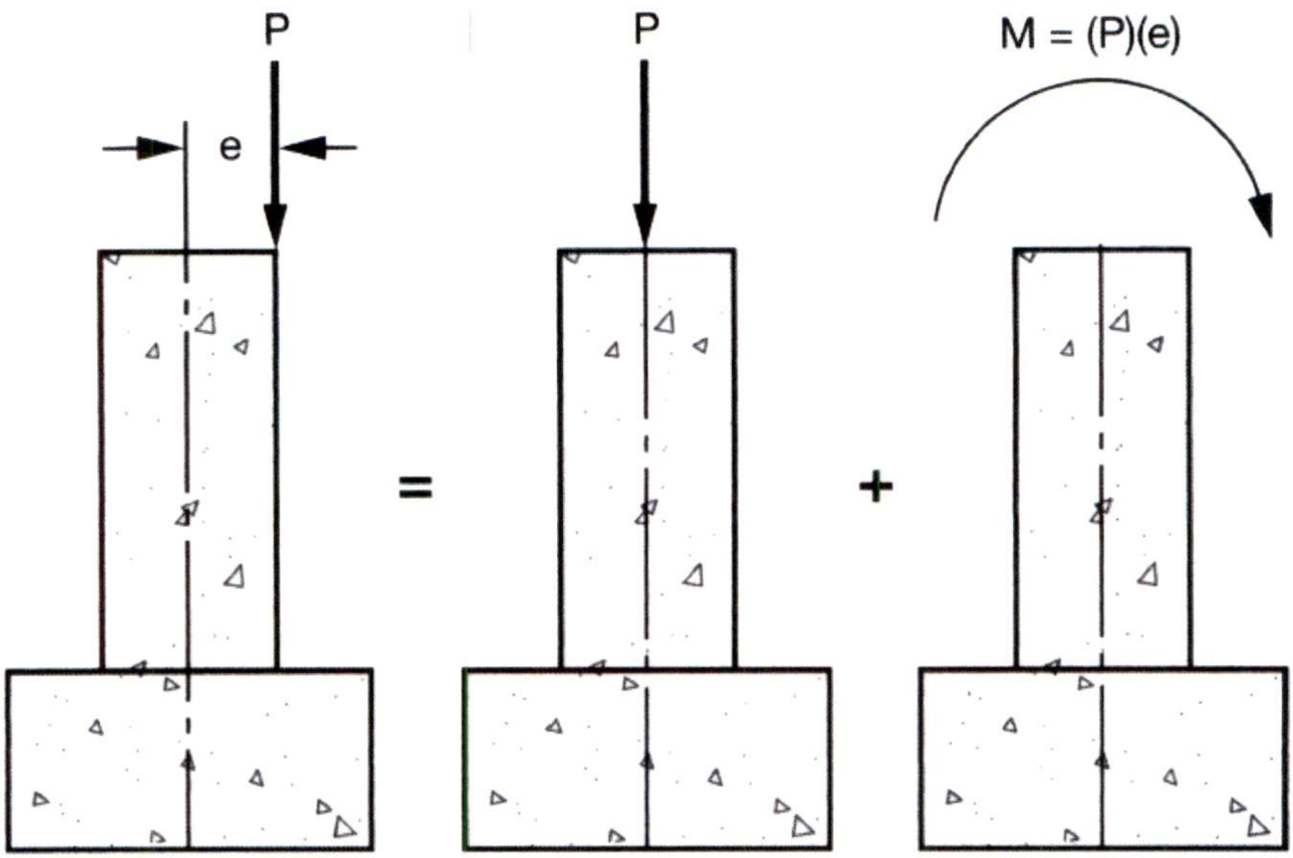
$$\sigma_B = \frac{-P}{A} + \frac{M}{S} = \frac{-P}{bh} + \frac{Pe}{hb^2/6} = 0$$

$$e = \frac{b}{6}$$

Along Thickness, same approach

$$e = h/6$$

Eccentrically Loaded Pier or Column



A common case of combined stress due to eccentric loads is the case of an eccentrically loaded column.

- This column carries an axial force, P, that must be resisted by the column developing a compressive stress. The compressive stress would be uniform over the entire resisting area and calculated using the direct stress formula from Table 18-1.

<u>Direct Normal Stress</u>

$$\sigma = \frac{P}{A}$$

- Because the load is not applied through the center of the column, it will cause bending about the shape's neutral axis --- typically the centroidal axis.

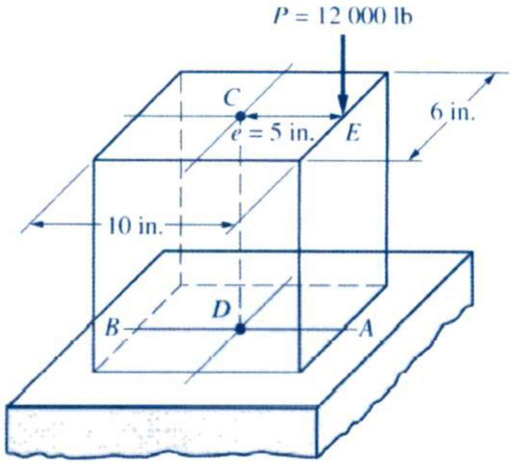
$$M = Pe$$

where,

e = eccentric distance or eccentricity

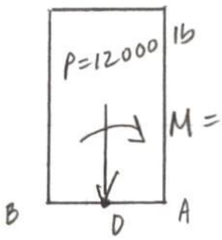
Example 18-5

A full-size, 6 in. X 10 in. rectangular short timber post carries an eccentrically placed axial load P = 12 000 lb as shown. Determine the normal stresses at points A and B.



Solution.

Equivalent Force System



$M = Pe = 12\,000 \text{ lb} (5 \text{ in}) = 60{,}000 \text{ lb}\cdot\text{in}$

$$S = \frac{bh^2}{6} = \frac{(6\text{ in})(10\text{ in})^2}{6} = 100 \text{ in.}^3 \qquad A = 6\text{ in}\,(10\text{ in}) = 60 \text{ in.}^2$$

Normal Stress at Point A

$$\sigma_A = -\frac{P}{A} - \frac{M}{S} = \frac{-12000 \text{ lb}}{60 \text{ in}^2} - \frac{60000 \text{ lb}\cdot\text{in}}{100 \text{ in}^3}$$

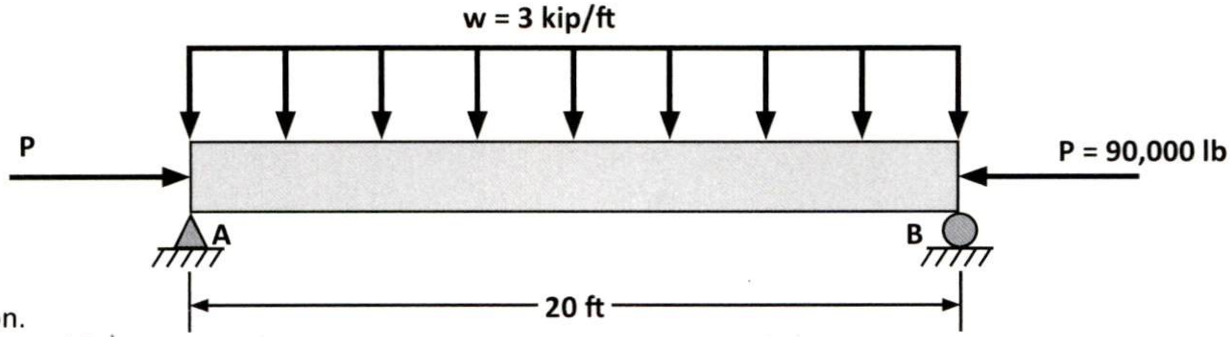
$$= -200 \text{ psi} - 600 \text{ psi} = \underline{-800 \text{ psi}} \text{ (C)}$$

Normal Stress at Point B

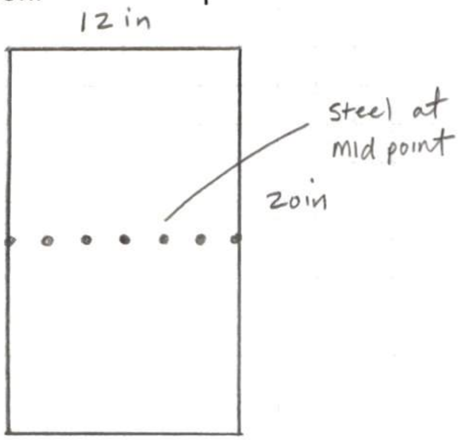
$$\sigma_B = -\frac{P}{A} + \frac{M}{S} = -200 \text{ psi} + 600 \text{ psi} = \underline{400 \text{ psi}} \text{ (T)}$$

Example 6 - No Eccentricity of the Axial Load

A rectangular beam measuring 12 in. wide by 20 in. deep is prestressed using steel cables located at the centroidal axis and delivering a compressive force of 90,000 lb to the end of the beam. If the beam is loaded as shown below, calculate the combined stress distribution in the beam.



Solution.



$P = 90{,}000 \text{ lb}$

$w = 3 \text{ kip/ft}$

$A = 12\text{in}(20\text{in}) = 240 \text{ in.}^2$

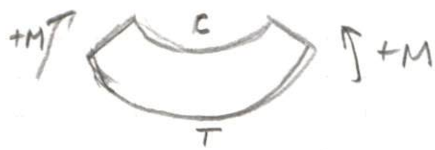
$S = \dfrac{bh^2}{6} = \dfrac{12\text{in}(20\text{in})^2}{6} = 800 \text{ in}^3$

$L = 20 \text{ ft}$

Normal Stress from Prestress

$$\sigma_1 = -\frac{P}{A} = -\frac{90{,}000 \text{ lb}}{240 \text{ in}^2} = -375 \text{ psi (c)}$$

Normal Stress from Bending (uniform Load)



From Table 13-1, case 4 (at midspan)

$$M_{MAX} = \frac{wL^2}{8} = \frac{3\text{kip/ft}\,(20\text{ft})^2}{8} = 150 \text{ kip}\cdot\text{ft}\left(\frac{12\text{in}}{\text{ft}}\right) = 1800 \text{ kip}\cdot\text{in}$$

$$\sigma_2 = \frac{M}{S} = \frac{1800 \text{ kip}\cdot\text{in}}{800 \text{ in}^3} = 2.25 \text{ ksi}\left(\frac{1000 \text{ lb}}{\text{kip}}\right) = 2250 \text{ psi}$$

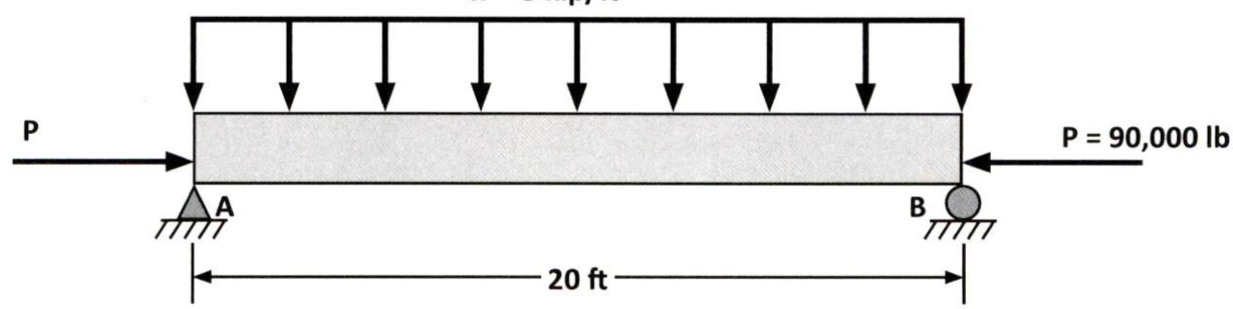
Combined Stresses

TOP OF BEAM $\quad \sigma_{MAX}^{(C)} = -375 \text{ psi} - 2250 \text{ psi} = -2625 \text{ psi (c)}$

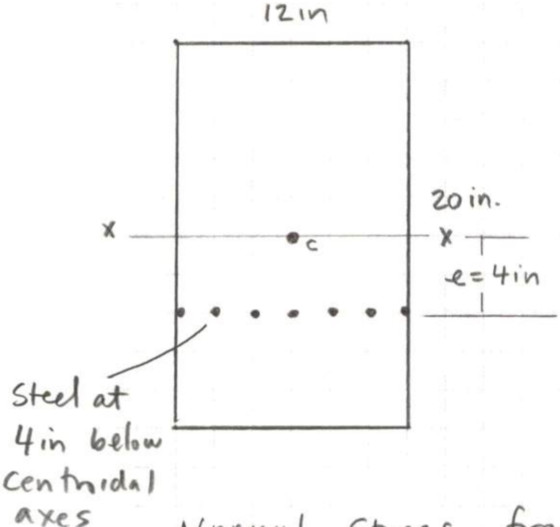
Bottom of Beam $\quad \sigma_{MAX}^{(T)} = -375 \text{ psi} + 2250 \text{ psi} = 1875 \text{ psi (T)}$

Example 6 - Eccentric Axial Load

A rectangular beam measuring 12 in. wide by 20 in. deep is prestressed using steel cables located 4 in. below the centroidal axis and delivers a compressive force of 90,000 lb to the end of the beam. Calculate the combined stress due to the prestressing effect. After it is erected on site it is subjected to an additional load of 3 kip per ft as shown below. Recalculate the combined stress in the beam after the beam is subjected to this loading.



Solution.



$P = 90{,}000 \text{ lb}$

$w = 3 \text{ kip/ft}$

$L = 20 \text{ ft}$

$A = 12\text{in}(20\text{in}) = 240 \text{ in.}^2$

$$S = \frac{bh^2}{6} = \frac{12\text{in}(20\text{in})^2}{6} = 800 \text{ in.}^3$$

Normal Stress from Prestress

The prestressing force is a compressive force that also causes bending because of its eccentricity.

Determine the Combined Stresses from Prestressing

Normal Stress from compressive force:

$$\sigma_1 = -\frac{P}{A} = -\frac{90{,}000 \text{ lb}}{240 \text{ in}^2} = -375 \text{ psi (c)}$$

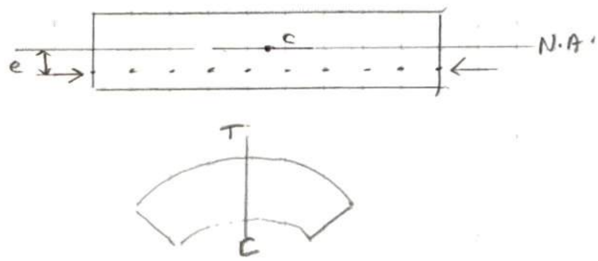
Normal stress from Bending:

$$M = Pe = 90{,}000 \text{ lb}(4 \text{ in}) = 360{,}000 \text{ lb}\cdot\text{in}$$

$$\sigma_2 = \frac{M}{S} = \frac{360{,}000 \text{ lb}\cdot\text{in}}{800 \text{ in.}^3} = 450 \text{ psi}$$

TOP OF Beam

Prestressing Below N.A.



$$\sigma = -\sigma_1 + \sigma_2$$

$$= -375\ psi + 450\ psi$$

$$= 75\ psi\ (T)$$

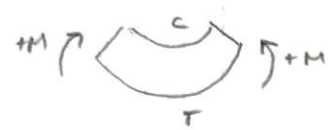
Bottom of Beam

$$\sigma = -\sigma_1 - \sigma_2$$

$$= -375\ psi - 450\ psi$$

$$= -825\ psi\ (C)$$

Normal stress from Bending - Uniform Load



$$M_{MAX} = \frac{wL^2}{8} = \frac{3\ kip/ft\ (20\ ft)^2}{8} = 150\ kip\cdot ft\left(\frac{12\ in}{ft}\right) = 1800\ kip\cdot in$$

$$\sigma_3 = \frac{M}{S} = \frac{1800\ kip\cdot in}{800\ in.^3} = 2.25\ ksi\left(\frac{1000\ lb}{kip}\right) = 2250\ psi$$

Combined Stresses

TOP OF BEAM

$$\sigma_{MAX} = -\sigma_1 + \sigma_2 - \sigma_3 = -375\ psi + 450\ psi - 2250\ psi = -2175\ psi$$

Bottom of Beam

$$\sigma_{MAX} = -\sigma_2 - \sigma_2 + \sigma_3 = -375\ psi - 450\ psi + 2250\ psi = +1425\ psi$$

$$\boxed{\begin{array}{l} \sigma_{MAX}^{(C)} = -2175\ psi\ (C) \\ \sigma_{MAX}^{(T)} = 1425\ psi\ (T) \end{array}}$$