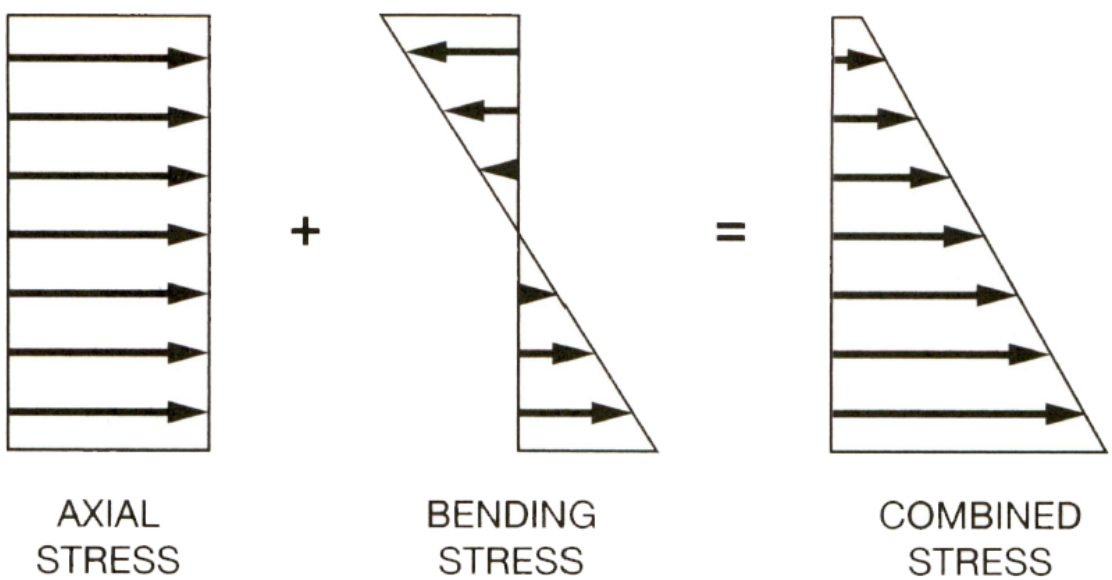
Chapter 18 - Combined Stresses
Reading: Chapter 18 — Pages 653-700

18-1
Introduction

- Formulas for calculating the stresses in members subjected to axial, bending, and shear stresses have been developed in the previous chapters in a separate and individual manner.
- The formulas were derived based on the assumption that the stresses were caused by only one type of loading, and the maximum stress in the member was within the elastic limit of the material, within which stress was proportional to strain.
- In many engineering applications, more than one type of loading may be applied to a member and the member may be subjected to more than one type of stress. Therefore, a technique is needed for finding the combined stress in a member due to several types of loading.
- The method of superposition is used to determine the combined stresses caused by two or more types of loading. Using this method and the fundamental formulas in Table 18-1, the same type of stresses caused by each loading are determined separately. The algebraic sum of these stresses gives the combined stresses caused by all the loadings acting simultaneously. The method of superposition is valid only if the maximum stress is within the elastic limit of the material and if the deformations are small.



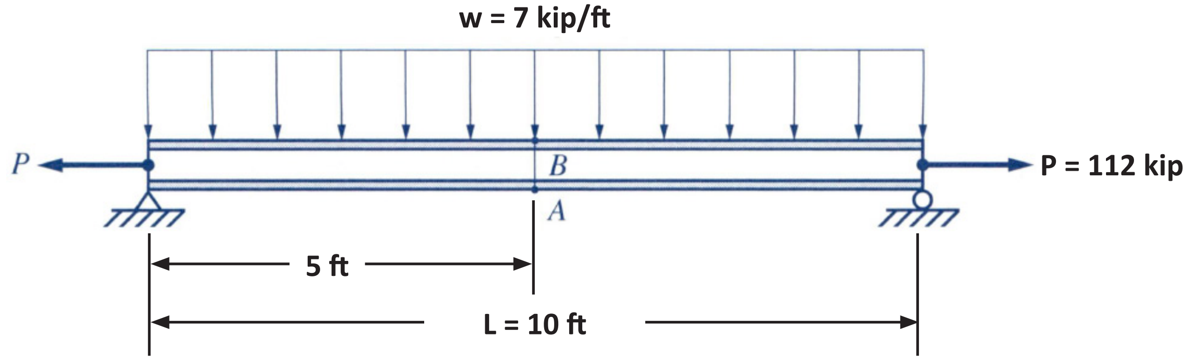
Principle of Super Position with Combined Stress

TABLE 18–1 List of the Fundamental Formulas

Type of Load	*Type of Stress*	*Formula*	*Equation Number*
Axial load	Direct normal stress	$\sigma = \frac{P}{A}$	(9–1)
Internal pressure in thin-walled vessels	Circumferential stress	$\sigma_c = \frac{Pr_i}{t}$	(9–16)
	Longitudinal stress	$\sigma_l = \frac{Pr_i}{2t}$	(9–17)
Beam bending load	Flexural stress	$\sigma = \frac{My}{I}$	(14–3)
		$\sigma_{max} = \frac{Mc}{I}$	(14–2)
		$\sigma_{max} = \frac{M}{S}$	(14–7)
Direct shear load	Direct shear stress	$\tau_{avg} = \frac{P}{A}$	(9–4)
Torque in circular shaft	Torsional shear stress	$\tau = \frac{T\rho}{J}$	(12–2)
		$\tau_{max} = \frac{Tc}{J}$	(12–1)
Beam shear force	Beam shear stress	$\tau = \frac{VQ}{It}$	(14–10)
	Maximum shear stress in rectangular section	$\tau_{max} = 1.5\frac{V}{A}$	(14–11)
	Maximum shear stress in circular section	$\tau_{max} = \frac{4V}{3A}$	(14–12)

Example 18-1 [Converted to U.S. Units]

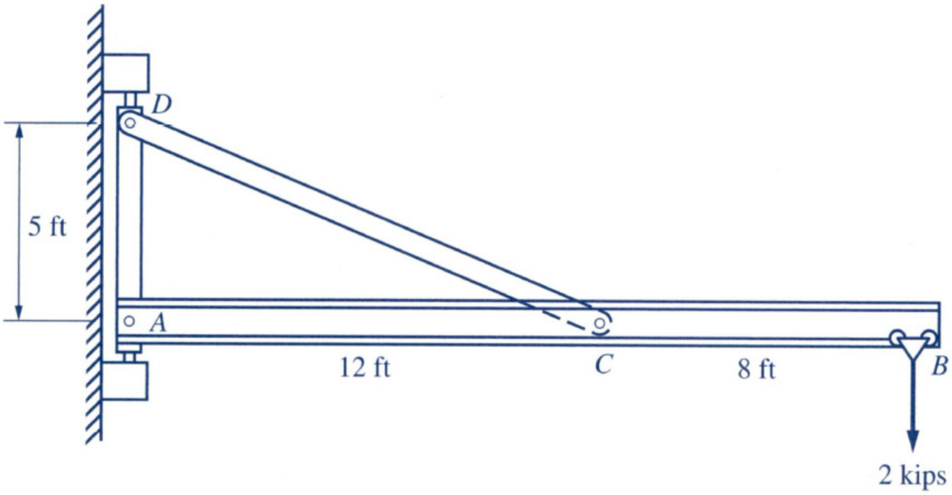
The wide-flange shape W14 x 68 is used as a simple beam of 10-ft span. The beam is subjected to a uniform load w of 7 kip/ft (including the weight of the beam) and an axial tensile force P of 112 kips. Determine the normal stresses at points A and B, and plot the normal stress variation between A and B.



Solution.

Example 18-2

A crane with a swinging arm is designed to hoist a maximum weight of 2 kips. If the allowable compressive stress is 13 ksi, select a W shape for the arm AB.



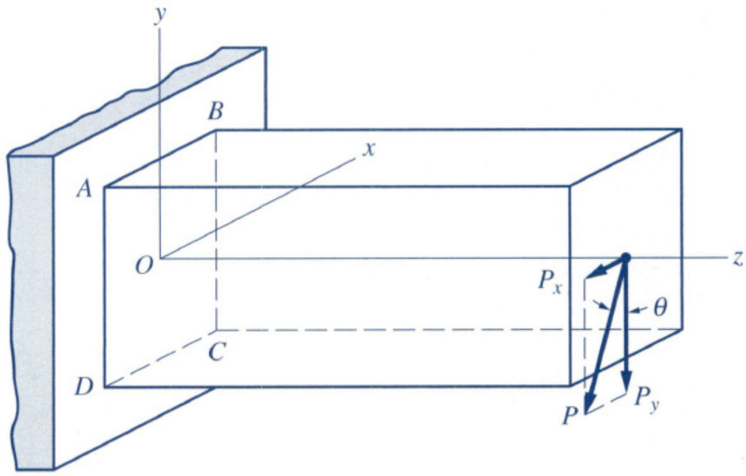
Solution.

18-3
Biaxial Bending

- Problems can arise when a load is inclined at an angle with respect to the vertical plane of symmetry of the beam.
- Load P can be resolved into its horizontal (P_x) and vertical (P_y) components, in the directions of the two axes of symmetry at the section on the free end.

- P_y causes bending about the horizontal axis
- P_x causes bending about the vertical axis

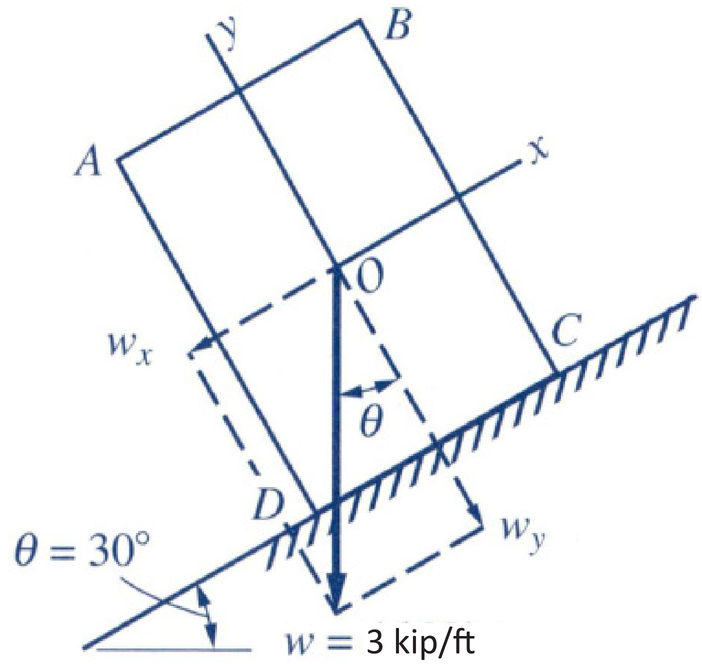
Biaxial Bending



- Either type of bending causes normal stresses along the longitudinal direction and superposition can be applied.
- The bending about each axis is calculated separately and the results are added algebraically.

Example 18-3 [Converted to U.S. Units]

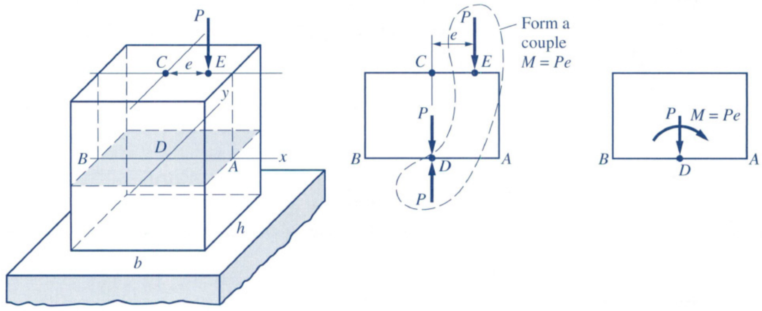
A simply supported timber beam of 10-ft span has a rectangular cross-section with a nominal size of 6-in x 8-in. The beam carries a uniform load w of 3 kip/ft and is supported at the ends in the tilted position shown in Fig. E18-3. Determine the maximum flexural stresses in the beam.



Solution.

18-4
Eccentrically Loaded Members

- Another case of combined stress due to axial and bending effects arises when an axial load acts away from the centroidal axis of a member. Such a load is, therefore, referred to as an eccentric load.
- Eccentric loads, by their nature, subject the member to axial stress and to a bending stress. The bending stress develops from the moment caused by the load, multiplied by the eccentric distance or eccentricity.
- The method of superposition can be applied to a short compression member that has small deflections and will not buckle under compressive loads.



An eccentric axial load can be replaced by a concentric force and a couple.

The original downward force at E and the upward force at D form a couple M = Pe. The system is thus reduced to a concentric force P at the centroid D and a couple M = Pe, as shown above.

The combined stresses created by the axial force and bending moment can be determined in the same way as in section 18-2.

The axial force produces a uniform compressive stress throughout the section.
The bending moment produces maximum compressive stress at A and maximum tensile stress at B.

By superposition, the normal stresses at A and B are:

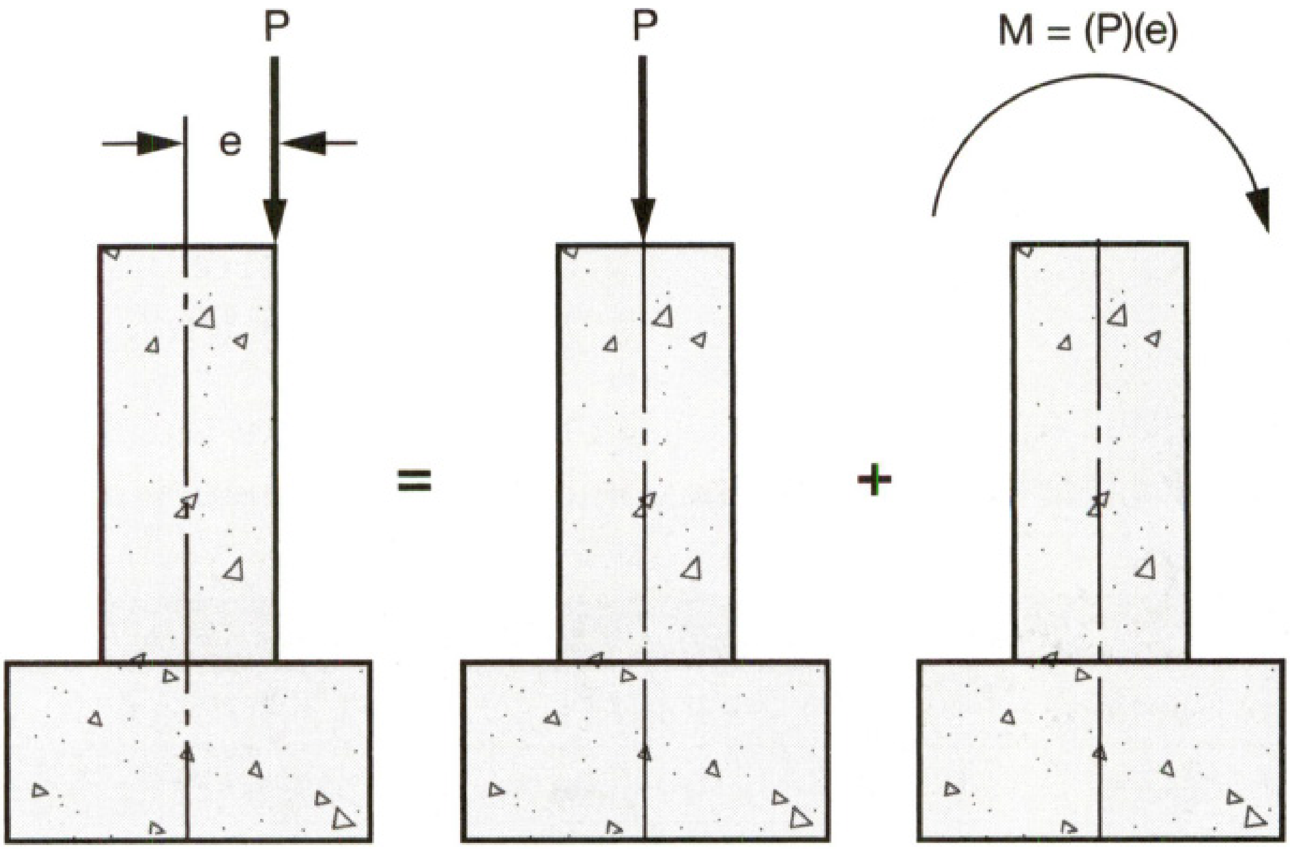
$$\sigma_A = \frac{-P}{A} - \frac{M}{S} \quad (18\text{-}1)$$ (Normal stress at A is always compressive)

$$\sigma_B = \frac{-P}{A} + \frac{M}{S} \quad (18\text{-}2)$$ (Normal stress at B may be compressive, tensile, or zero, depends on e)

Maximum Eccentricty
For some materials that are weak in tension, keeping the eccentricty of a compressive load to a certain maximum limit so that no tensile stress develops anywhere in the member may be important.

Limit of Eccentricty for Solid Rectangular Section

Eccentrically Loaded Pier or Column



A common case of combined stress due to eccentric loads is the case of an eccentrically loaded column.

- This column carries an axial force, P, that must be resisted by the column developing a compressive stress. The compressive stress would be uniform over the entire resisting area and calculated using the direct stress formula from Table 18-1.

<u>Direct Normal Stress</u>

$$\sigma = \frac{P}{A}$$

- Because the load is not applied through the center of the column, it will cause bending about the shape's neutral axis --- typically the centroidal axis.

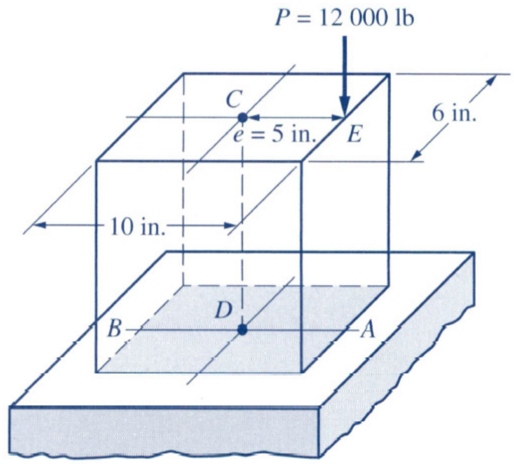
$$M = Pe$$

where,

e = eccentric distance or eccentricity

Example 18-5

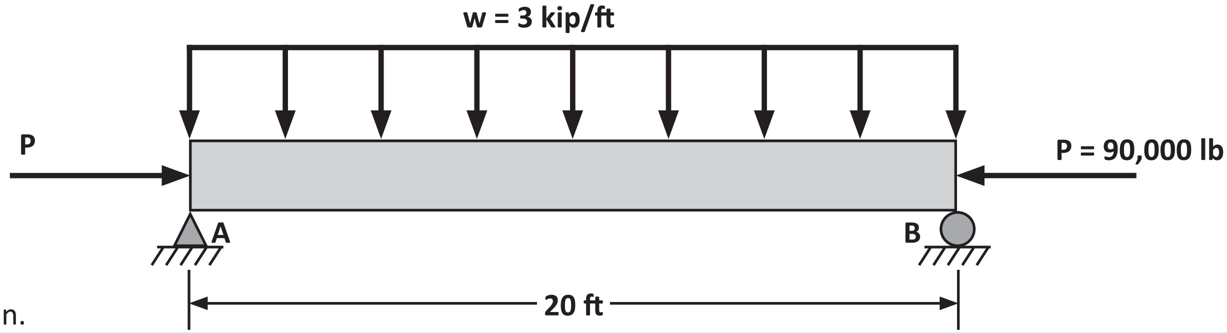
A full-size, 6 in. X 10 in. rectangular short timber post carries an eccentrically placed axial load P = 12 000 lb as shown. Determine the normal stresses at points A and B.



Solution.

Example 6 - No Eccentricity of the Axial Load

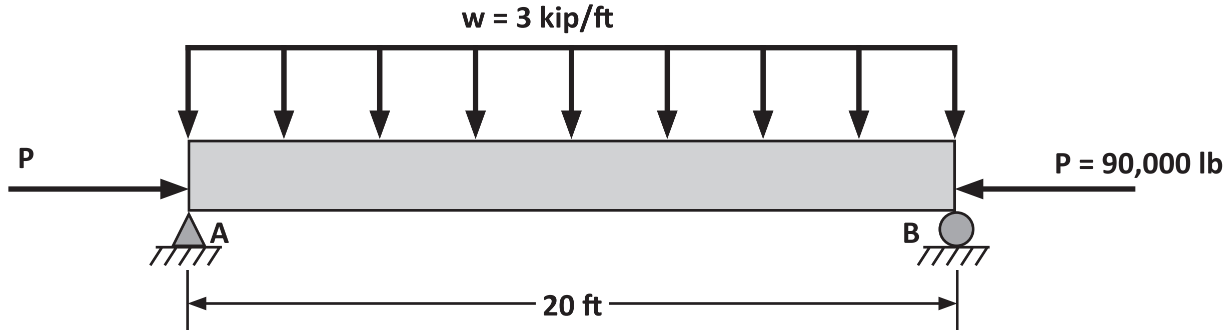
A rectangular beam measuring 12 in. wide by 20 in. deep is prestressed using steel cables located at the centroidal axis and delivering a compressive force of 90,000 lb to the end of the beam. If the beam is loaded as shown below, calculate the combined stress distribution in the beam.



Solution.

Example 6 - Eccentric Axial Load

A rectangular beam measuring 12 in. wide by 20 in. deep is prestressed using steel cables located 4 in. below the centroidal axis and delivers a compressive force of 90,000 lb to the end of the beam. Calculate the combined stress due to the prestressing effect. After it is erected on site it is subjected to an additional load of 3 kip per ft as shown below. Recalculate the combined stress in the beam after the beam is subjected to this loading.



Solution.