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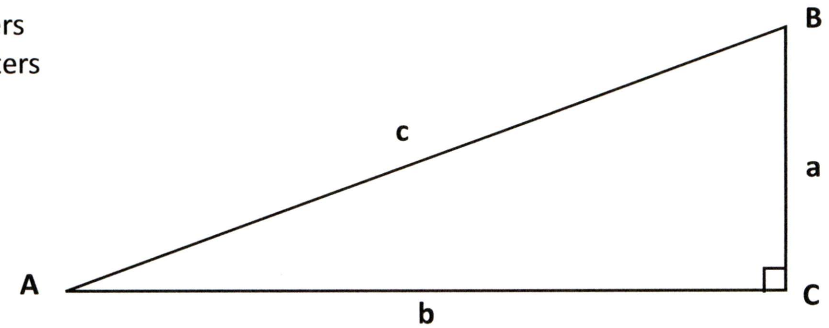
A Brief Review of Mathematics

Mathematics Required for Statics and Strengths:
Arithmetic
Algebra
Geometry
Trigonometry

Right Triangle -Three-sided closed figure that has a right angle (angle equals 90°)

Angles - Capital Letters
Sides - lowercase letters



Sum of the interior angles of a triangle is 180°

$$A + B + C = 180^\circ$$

$$C = 90^\circ$$

$$A + B = 90^\circ$$

Pythagorean Theorem

$$c^2 = a^2 + b^2$$

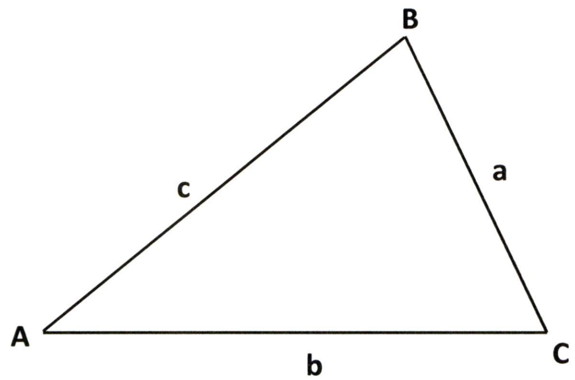
Trigonometry of Right Triangles

$$\text{Sin } A = \frac{\text{Opposite}}{\text{Hypotenuse}} = \frac{a}{c}$$

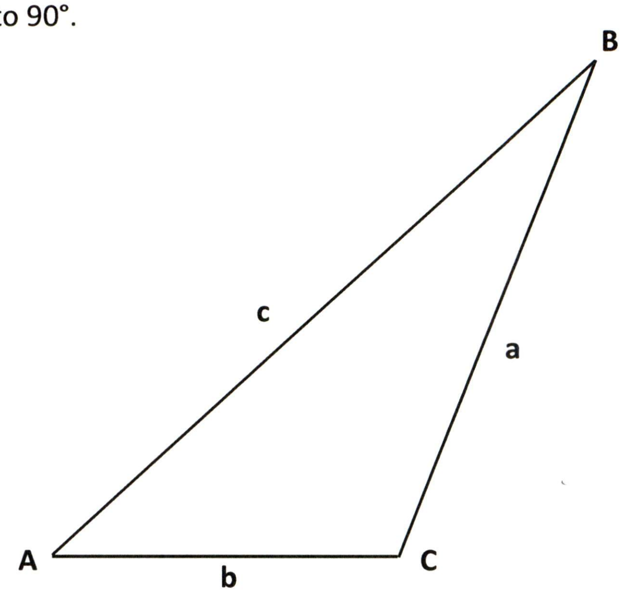
$$\text{Cos } A = \frac{\text{Adjacent}}{\text{Hypotenuse}} = \frac{b}{c}$$

$$\text{Tan } A = \frac{\text{Opposite}}{\text{Adjacent}} = \frac{a}{b}$$

Oblique Triangle - None of the interior angles are equal to 90°.



Acute Oblique Triangle



Obtuse Oblique Triangle

Sum of the interior angles of a triangle is 180°

$$A + B + C = 180°$$

Trigonometry of Oblique Triangles

Law of Sines. The ratio of any side of a triangle to the sine function of its opposite angle is a constant:

$$\frac{a}{\text{Sin } A} = \frac{b}{\text{Sin } B} = \frac{c}{\text{Sin } C}$$

conversely,

$$\frac{\text{Sin } A}{a} = \frac{\text{Sin } B}{b} = \frac{\text{Sin } C}{c}$$

Law of Cosines. The square of any side of a triangle is equal to the sum of the squares of the other two sides minus twice the product of the two sides multiplied by the cosine function of the angle between them:

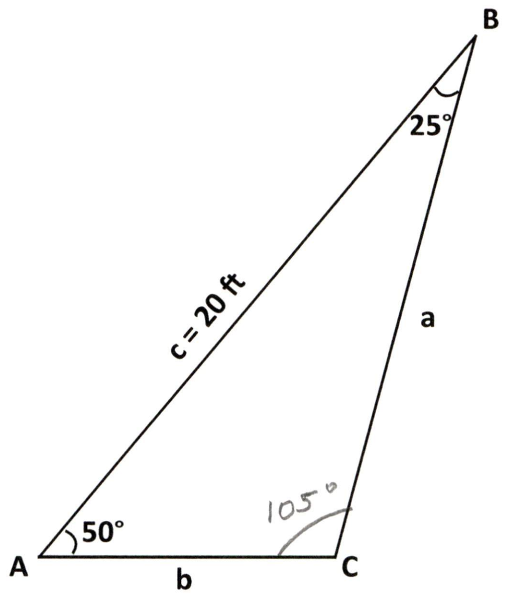
$$a^2 = b^2 + c^2 - 2bc \text{ Cos } A$$

$$b^2 = a^2 + c^2 - 2ac \text{ Cos } B$$

$$c^2 = a^2 + b^2 - 2ab \text{ Cos } C$$

How do you determine which oblique triangle trig identity to use?

Example. For the oblique triangle shown, solve for the lengths of side a and side b and for angle C.



Solution.

$A + B + C = 180°$

$50° + 25° + C = 180°$

$C = 180° - 75° = \underline{\underline{105°}}$

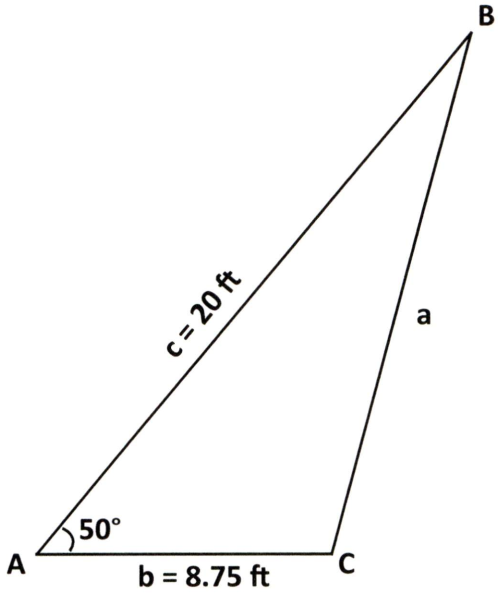
$\boxed{ASA}$ LAW OF SINES

$$\frac{a}{\sin 50°} = \frac{b}{\sin 25°} = \frac{20 \text{ ft}}{\sin 105°}$$

$$a = \frac{20 \text{ ft} \sin 50°}{\sin 105°} = \underline{\underline{15.9 \text{ ft}}}$$

$$b = \frac{20 \text{ ft} \sin 25°}{\sin 105°} = \underline{\underline{8.75 \text{ ft}}}$$

Example. For the oblique triangle shown, solve for the lengths of side a and for angles B & C.



Solution.

$\boxed{\text{SAS}}$ LAW OF COSINES

$$a^2 = 8.75\,ft^2 + 20\,ft^2 - 2(8.75\,ft)(20\,ft)\cos 50^\circ$$

$$= 476.5625\,ft^2 - 224.9757$$

$$= 251.5868$$

$$a = \sqrt{251.5868} = 15.9\,ft$$

LAW OF SINES

$$\frac{\sin B}{8.75\,ft} = \frac{\sin 50^\circ}{15.9\,ft}$$

$$B = \sin^{-1}\left(\frac{8.75\,ft\,\sin 50^\circ}{15.9\,ft}\right) = 25^\circ$$

$$C = 180^\circ - 50^\circ - 25^\circ = 105^\circ$$

Simultaneous Equations

Solving two simultaneous linear equations containing two unknowns sometimes occurs in two-dimensional equilibrium problems.

Methods for Solving Simultaneous Linear Equations

Method 1. Elimination by Substitution

Method 2: Elimination by Addition and Subtraction

Method 3: Cramer's Rule

Example. Solve the following equations for x and y by using Elimination by Substitution.

$$4x + y = 10 \qquad (1)$$
$$3x - 5y = 19 \qquad (2)$$

Solution.

solve (1) for x in terms of y

$$4x + y = 10$$
$$4x = 10 - y$$
$$x = \frac{10-y}{4} \qquad (3)$$

Subst. (3) into (2)

$$3\left[\frac{10-y}{4}\right] - 5y = 19$$
$$4\left[3\left(\frac{10-y}{4}\right) - 5y\right] = 19 \quad (4)$$
$$3(10-y) - 20y = 76$$
$$30 - 3y - 20y = 76$$
$$-23y = 46$$
$$y = \frac{46}{-23} = \underline{\underline{-2}}$$

From (3) $x = \frac{10 - -2}{4} = \frac{12}{4} = \underline{\underline{3}}$

Check, $4(3) + -2 = 10$
$$12 - 2 = 10$$
$$10 = 10 \checkmark$$

Example. Solve the following equations for x and y by using Elimination by Addition & Subtraction.

$$4x + y = 10 \qquad (1)$$
$$3x - 5y = 19 \qquad (2)$$

Solution.

multiply (1) by 3 and (2) by 4 and Subtract

$$\begin{array}{rl} & 12x + 3y = 30 \\ - & 12x - 20y = 76 \\ \hline & 23y = -46 \end{array}$$

$$y = \frac{-46}{23} = \underline{\underline{-2}}$$

Subst. into (1) or (2)

From (1)

$$4x + -2 = 10$$
$$4x = 12$$
$$x = \frac{12}{4} = \underline{\underline{3}}$$

Check,

$$3(3) - 5(-2) = 19$$
$$9 + 10 = 19$$
$$19 = 19 \checkmark$$

Matrix Math

Matrix – array of numbers arranged in row and column format.

Systems of equations of any order can be expressed in Matrix format.
Square Matrix – Number of rows equals the number of columns.

$$4x + y = 10 \qquad (1)$$
$$3x - 5y = 19 \qquad (2)$$

$$\begin{bmatrix} 4 & 1 \\ 3 & -5 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 10 \\ 19 \end{bmatrix}$$

Matrix Form

Determinant of a Matrix
The determinant of a matrix is a number.

Determinant of a matrix of order 2:

$$\text{Det A} = \begin{vmatrix} a_1 & b_1 \\ a_2 & b_2 \end{vmatrix} = a_1b_2 - a_2b_1$$

Cramer's Rule for Two Simultaneous Linear Equations with Two Unknowns

$a_1x + b_1y = k_1$
$a_2x + b_2y = k_2$

$$x = \frac{Dx}{D} \qquad y = \frac{Dy}{D}$$

where,

$$D = \begin{vmatrix} a_1 & b_1 \\ a_2 & b_2 \end{vmatrix} \qquad Dx = \begin{vmatrix} k_1 & b_1 \\ k_2 & b_2 \end{vmatrix} \qquad Dy = \begin{vmatrix} a_1 & k_1 \\ a_2 & k_2 \end{vmatrix}$$

Example. Solve the following equations for x and y by using Cramer's Rule.

$$4x + y = 10 \qquad (1)$$
$$3x - 5y = 19 \qquad (2)$$

Solution.

$$\begin{bmatrix} 4 & 1 \\ 3 & -5 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 10 \\ 19 \end{bmatrix}$$

$$D = \begin{vmatrix} 4 & 1 \\ 3 & -5 \end{vmatrix} = -20 - 3 = -23$$

$$D_x = \begin{vmatrix} 10 & 1 \\ 19 & -5 \end{vmatrix} = -50 - 19 = -69$$

$$D_y = \begin{vmatrix} 4 & 10 \\ 3 & 19 \end{vmatrix} = 76 - 30 = 46$$

$$x = \frac{D_x}{D} = \frac{-69}{-23} = \underline{\underline{3}}$$

$$y = \frac{D_y}{D} = \frac{46}{-23} = \underline{\underline{-2}}$$

For Cramer's Rule for solving 3 equations and 3 unknowns see example in textbook, Pg. 26-29.

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General Procedures for Problem Solution

Extensive applications of statics and strength of materials are based on a few simple principles. The most effective way of learning this subject is to solve problems of different levels of complexity. The following general procedure is helpful:

1. Read the problem carefully. Identify the given data and the unknown quantities to be determined.
2. Make a neat sketch showing all the quantities involved. For some problems, it may be helpful to tabulate the given data and the computed results.
3. Apply the relevant principles and express the physical conditions in mathematical form. The solution must be based on the principles and theorems presented in the text and must be executed in a logical manner.
4. The equations obtained must be dimensionally homogeneous. Values in consistent units must be used for substitution. The answer obtained must be rounded off to the proper degree of accuracy or precision.
5. Use your common sense and judgment to determine if the answer obtained is reasonable. In some problems, there are conditions in which answers can be checked. If such conditions are available, always use them to check the answers.
6. The engineering profession requires work that meets high standards. Students preparing to enter an engineering career must present their work in a neat and organized fashion.

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Summary

Forces. Mechanics is a physical science that studies the effects of forces. Forces are vector quantities. Vector quantities are characterized by a magnitude, a point of application, and a direction.

Types of Forces. Forces can be applied on a body by direct contact or through remote action. Forces can be concentrated at a point or distributed along a length, over an area, or throughout the entire body. External forces are exerted on the body by another body. Internal forces are the resisting forces within a body.

Types of Force Systems. Force systems can be classified into the following three types, depending on whether they are coplanar or spatial, concurrent or nonconcurrent.

1. Concurrent-coplanar force system
2. Nonconcurrent-coplanar force system
3. Spatial force system

Newton's Three Laws. These three laws form the foundation for the study of Newtonian mechanics. The first law deals with conditions for equilibrium of a particle and thereby lays the foundation for the study of statics. The second law provides the basic formulation for the study of dynamics. The third law provides the basic understanding for the nature of action and reaction forces.

The Principle of Transmissibility. The point of application of a force may be placed anywhere along the line of action of the force without changing the external effects of the force. However, the line of action and the direction of a force must be well defined. For the internal effect or the deformation of a body, a force acting on the body must have a fixed point of application, and therefore the principle of superposition does not apply.

System of Units. Two systems of units are used in this book: the U.S. customary units and the SI units. The base units in the U.S. system are the foot, second, and pound. The base unit for force (or weight), the pound, is dependent on gravitational attraction; it is therefore a gravitational system. The base units in the SI system are the meter, second, and kilogram. The base unit for mass, the kilogram, is independent of gravitational attraction; it is therefore an absolute system.

Rules for Numerical Computations. Calculated results should always be rounded off according to the following rules:

Rule 1 When approximate numbers are multiplied or divided, the result is expressed to the same accuracy as the least accurate number.

Rule 2 When approximate numbers are added or subtracted, the result is expressed to the same precision as the least precise number.

Mathematics Used in Mechanics. Some fundamental mathematical skills are required of the student. For example, students must be able to perform elementary algebraic manipulations, solve a right triangle using the Pythagorean theorem and trigonometric functions, solve an oblique triangle using the law of sines and/or the law of cosines, and solve two or three simultaneous linear equations.

General Procedure for Problem Solution. Problems must be solved in a logical and orderly manner. Students must learn to analyze the problem carefully. Make necessary sketches and apply the relevant principles. Equations must be solved by using proper mathematical operations. Results should be checked against certain required conditions or judged to be reasonable using common sense. Work must be presented in a neat and organized fashion.