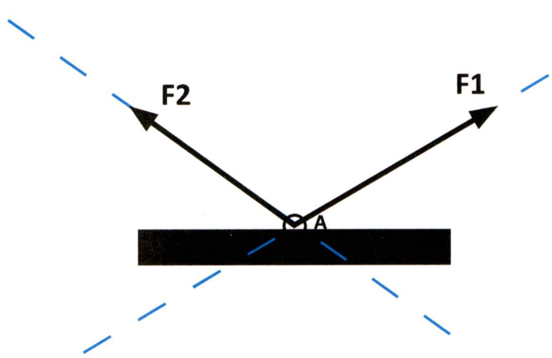
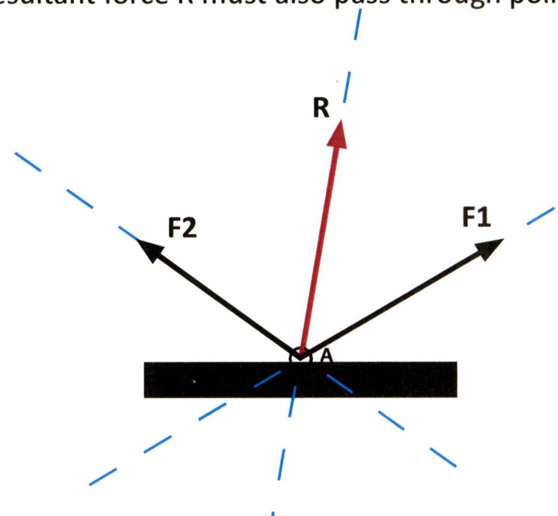
Chapter 2 - Resultant of Coplanar Force Systems
Reading: Chapter 2 - Pages 39-79

2-1
Introduction

Two systems of forces are said to be equivalent if they produce the same mechanical effect on a rigid body. A single force that is equivalent to a given force system is called the resultant of the force system.

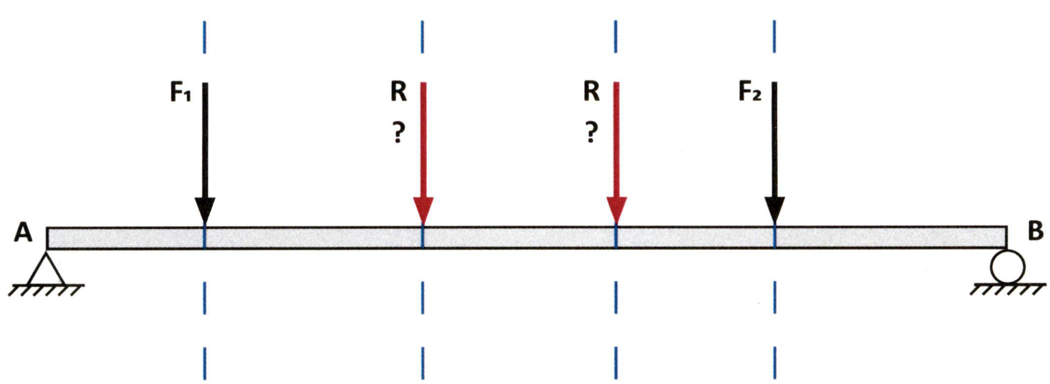
Resultant of a Concurrent Coplanar Force System
The forces F1 and F2 are acting on the rigid body at point A as shown. The line of action of each force are concurrent at point A. Therefore, the line of action of the resultant force R must also pass through point A.





<u>Methods for Determing the Resultant of a Concurrent Coplanar Force System</u>
Parallelogram Law
Triangle Rule
Graphically
Rectangular Components

Resultant of a Nonconcurrent Coplanar Force Systems
Any system of nonconcurrent coplanar forces can be replaced by a single resultant that is equivalent to the given force system.
The location of the line of action of the resultant is not immediately known.

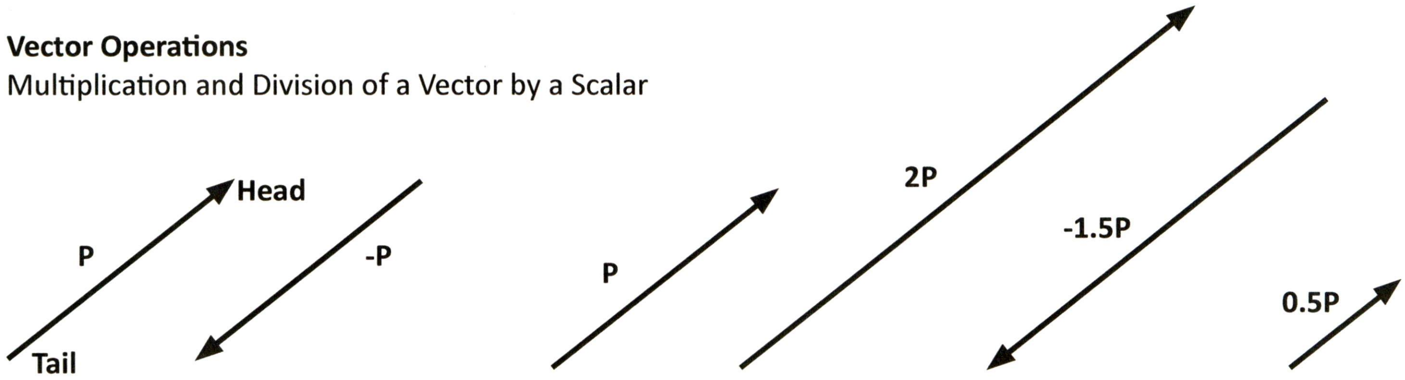


To determine the line of action of the resultant of a nonconcurrent coplanar force system, we will introduce the concepts of the moment of a force

2-2
Vector Representation

Scalars and Vectors
Definitions
Scalar: Any quantity possessing magnitude (size) only, such as mass, volume, temperature
Vector: Any quantity possessing both magnitude and direction, such as force, velocity, and momentum

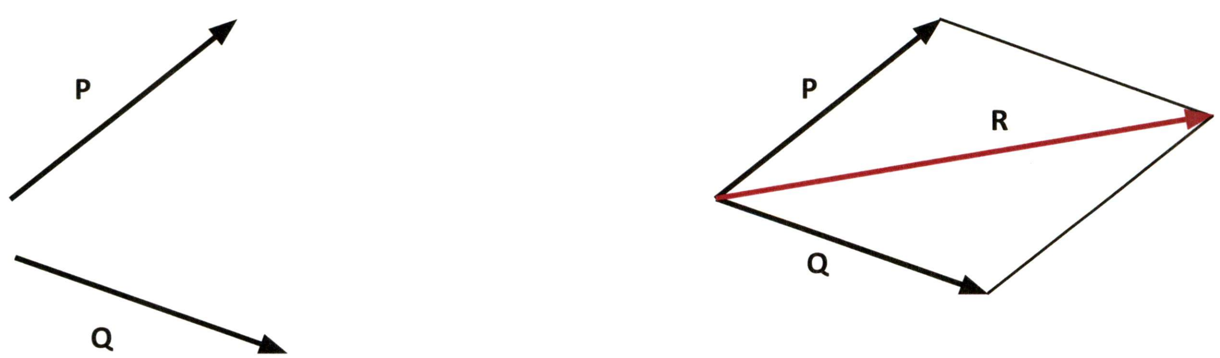
Vector Operations
Multiplication and Division of a Vector by a Scalar



Vector Addition
Two vectors **P** and **Q** may be added to form a "resultant" vector **R = P + Q**
Vector addition is commutative: **P + Q** = **Q + P**

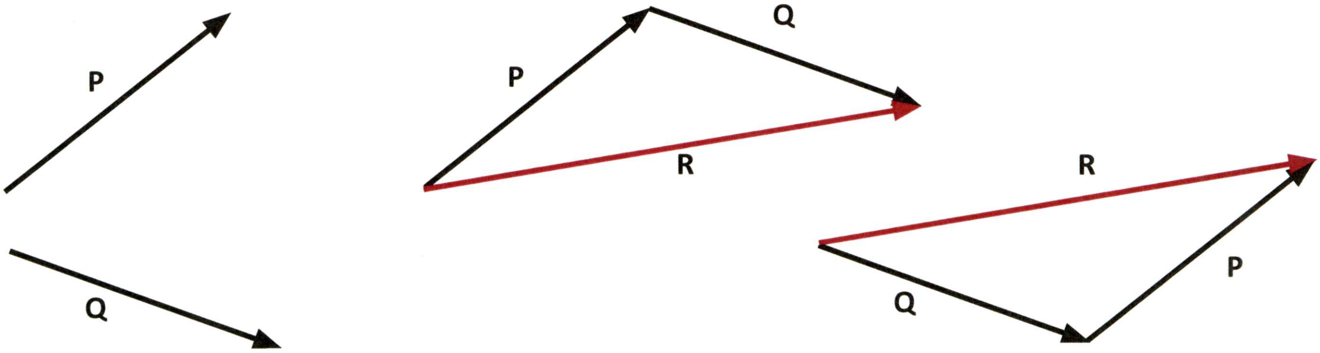
2-3
Resultant of Concurrent Forces

Method - Parallelogram Law
P and **Q** are joined at their tails.
Parallel lines drawn from the head of each vector intersect at a common point, forming the adjacent sides of a parallelogram.
Resultant **R** is the diagonal of the parallelogram, which extends from the tails of **P** and **Q** to the intersection of the lines.



Method - Triangle Rule

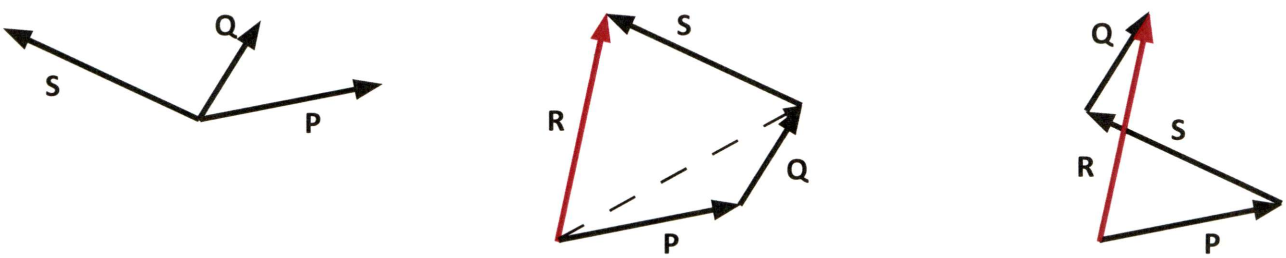
P and **Q** are added in a "head-to-tail" fashion by connecting the head of **P** to the tail of **Q**.
Resultant **R** extends from the tail of **P** to the head of **Q** forming a triangle.



Method - Polygon Rule

The sum of three or more concurrent coplanar vectors may be accomplished by adding two vectors successively.

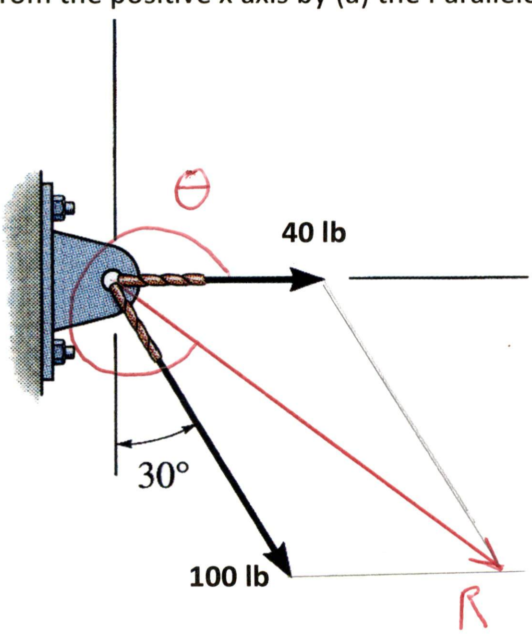
Vectors **P**, **Q**, and **S** can be added successively by first finding **P** + **Q**, then adding **S** to **P** + **Q**.
Forms a polygon called a force polygon.



or,

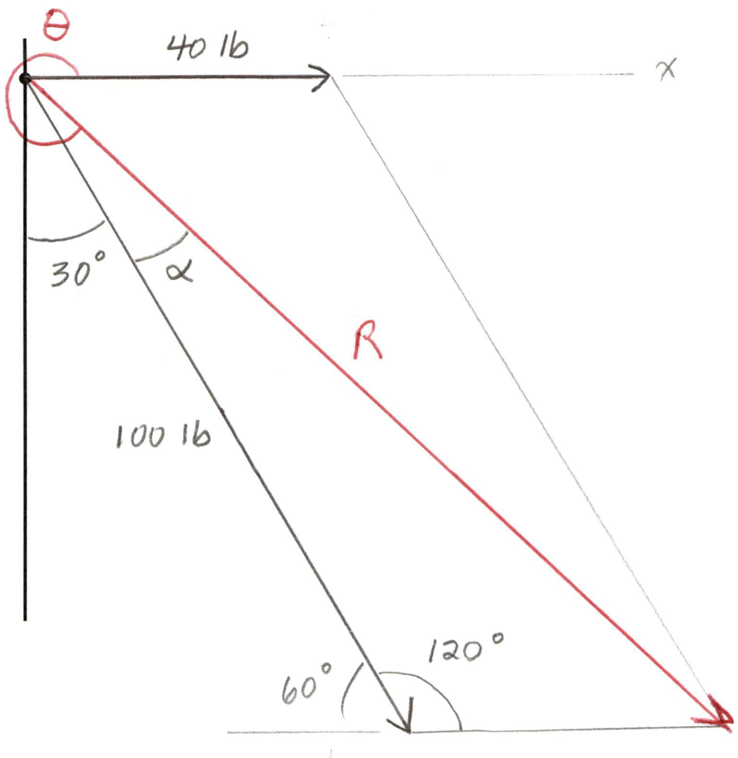
Vectors **P**, **Q**, and **S** can be added successively by first finding **P** + **S**, then adding **Q** to **P** + **S**.
Forms a polygon called a force polygon.

Example 1: Determine the magnitude of the resultant force and its direction measured counter-clockwise from the positive x axis by (a) the Parallelogram Law and (b) the Triangle Rule.



Find R and θ

Parallelogram Law (Tail-to-Tail)



SAS Law of Cosines

$$R = \sqrt{100\,lb^2 + 40\,lb^2 - 2(100\,lb)(40\,lb)\cos 120°}$$

$$= \sqrt{11{,}600\,lb^2 - -4000}$$

$$= \sqrt{15{,}600\,lb^2}$$

$$= 125\,lb \quad \text{(magnitude)}$$

Law of Sines

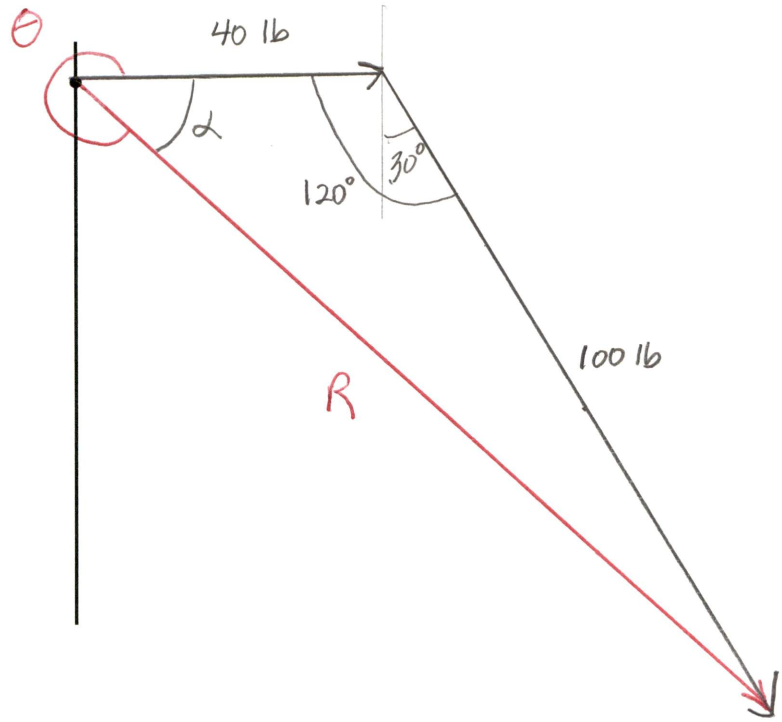
$$\frac{\sin\alpha}{40\,lb} = \frac{\sin 120°}{125\,lb}$$

$$\alpha = \sin^{-1}\left(\frac{40\,lb\,\sin 120°}{125\,lb}\right) = 16°$$

$$\theta = 270° + 30° + 16° = 316° \quad \text{(Direction)}$$

$$\boxed{R = 125\,lb \;@\; 316°}$$

Triangle Rule (Head – to – Tail)



SAS Law of Cosines

$$R = \sqrt{40\,lb^2 + 100\,lb^2 - 2(40\,lb)(100\,lb)\cos 120^\circ}$$

$$= 125\ lb \quad \text{(magnitude)}$$

Law of Sines

$$\frac{\sin\alpha}{100\,lb} = \frac{\sin 120^\circ}{125^\circ}$$

$$\alpha = \sin^{-1}\left(\frac{100\,lb\ \sin 120^\circ}{125\,lb}\right) = 44^\circ$$

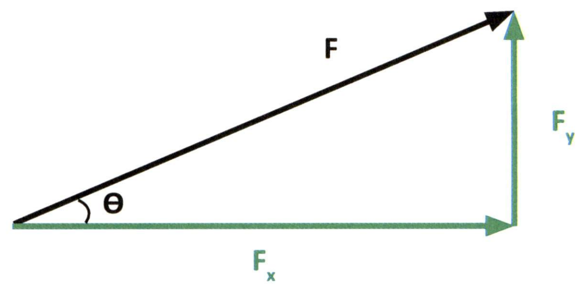
$$\theta = 270^\circ + 46^\circ = 316^\circ \quad \text{(Direction)}$$

$$\boxed{R = 125\,lb \angle 316^\circ}$$

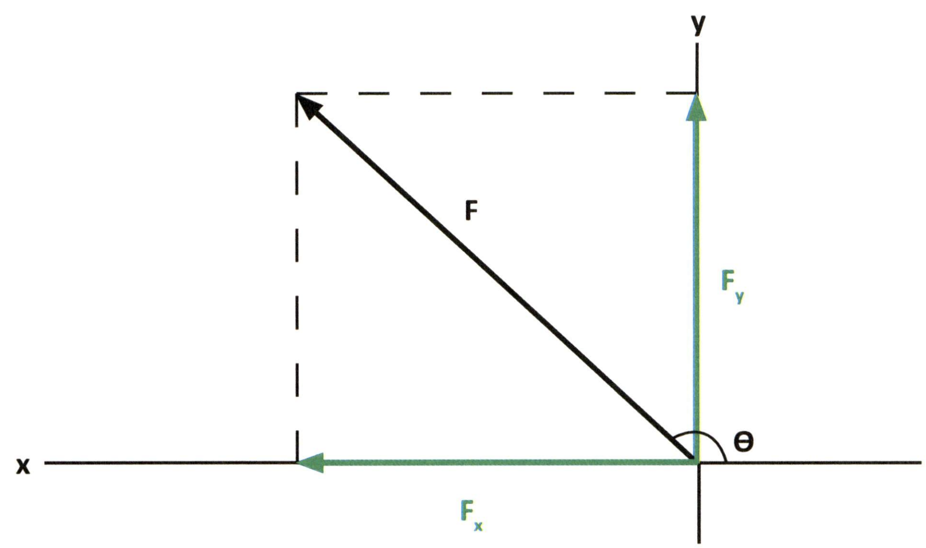
2-4
Rectangular Components

A vector can always be specified by its magnitude and direction, but it is often necessary or convenient to specify one by its rectangular components F_x and F_y (and F_z if the vector is three dimensional).

F_x the x component of the vector F, is the x coordinate of the head of F when the vector F is drawn with its tail at the origin of a coordinate system. The y component of F, F_y is similarly defined. F_x and F_y are illustrated in the figure below. Note that the rectangular components of a vector are real numbers that may be positive, zero, or negative.



The vector F shown above is a vector in the first quadrant.
The direction (angle ϴ) of a vector in the first quadrant is between 0° and 90°

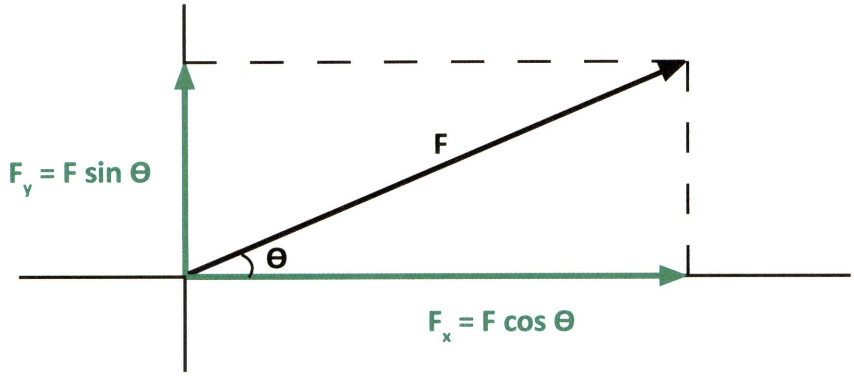


The vector F shown in the second diagram is a vector in the second quadrant.
The direction (angle ϴ) of a vector in the second quadrant is between >90° and 180°

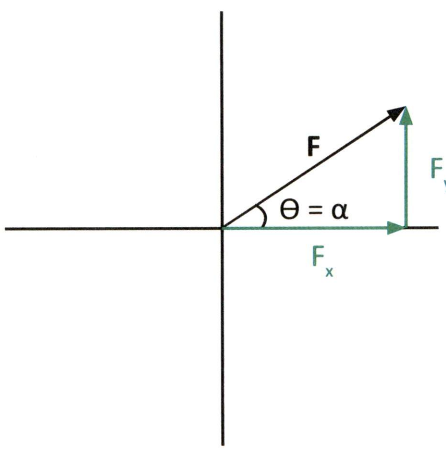
In the second quadrant x is negative and y is positive.

The **rectangular components** of vector F can be drawn beginning at the the tail of the vector. The x-component is drawn in the negative x direction (arrow to the left) and the y-component is drawn in the positive y direction (arrow upward). The dashed lines complete the "rectangle".

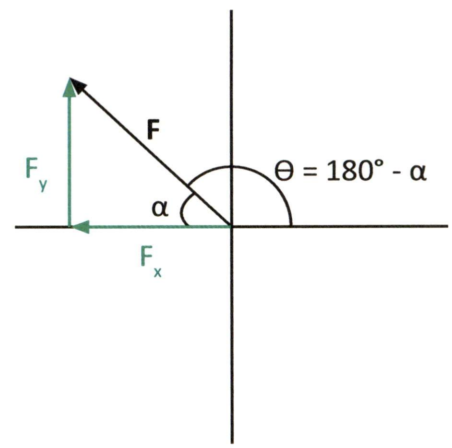
Any force can be resolved into its rectangular components Fx and Fy, which lie along the x and y axes respectively.



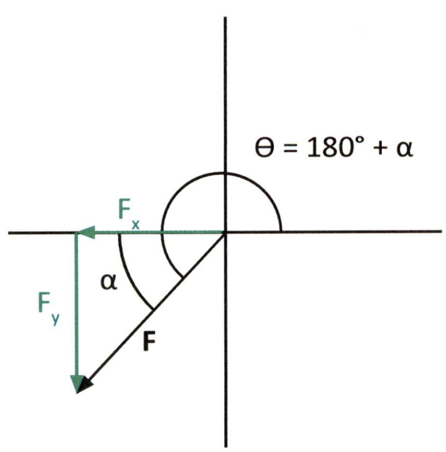
Magnitude	Direction
$F = \sqrt{F_x^{\,2} + F_y^{\,2}}$	$\theta = \alpha$ (First Quadrant) $\alpha = \tan^{-1}\left\|\dfrac{F_y}{F_x}\right\|$ (see below)



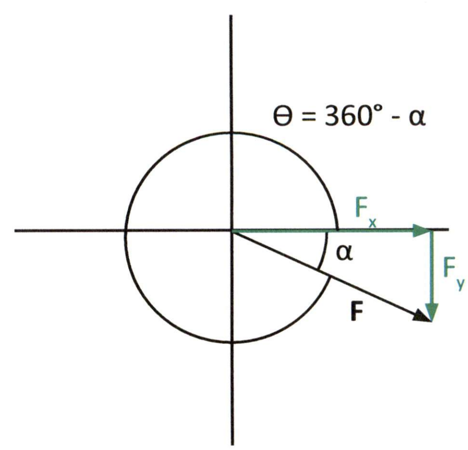
First Quadrant ϴ = α



Second Quadrant ϴ = 180° - α



Third Quadrant ϴ = 180° + α



Fourth Quadrant ϴ = 360° - α

2-5
Resultants by Rectangular Components

Using rectangular components, obtained by resolving forces in the system, the resultant can be obtained. The resultant force, just like any other force, can be thought of as being able to be resolved into x and y components R_x and R_y.

The magnitude of the components R_x and R_y is simply the sum of the x and y components of the forces in the system.

Magnitude

$$\left.\begin{array}{l} R_x = \sum F_x \\ \\ R_y = \sum F_y \end{array}\right\} R = \sqrt{R_x^2 + R_y^2}$$

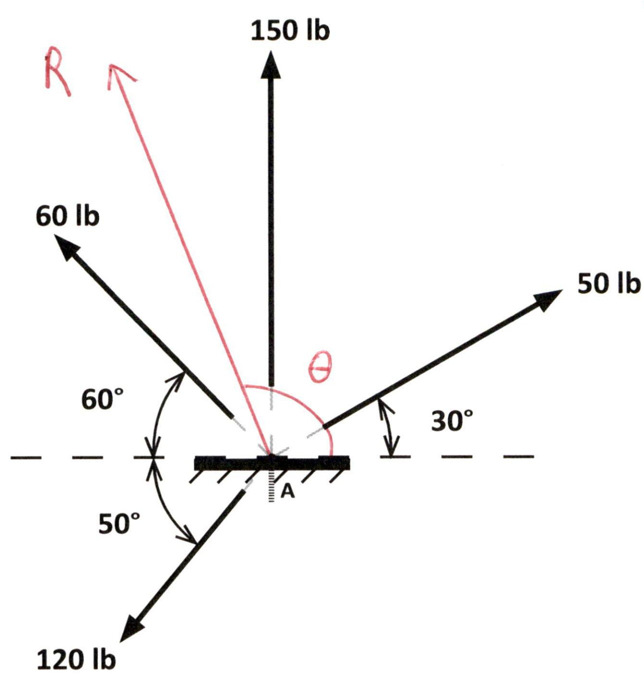
Direction

$$\alpha = \tan^{-1}\left|\frac{R_y}{R_x}\right|$$

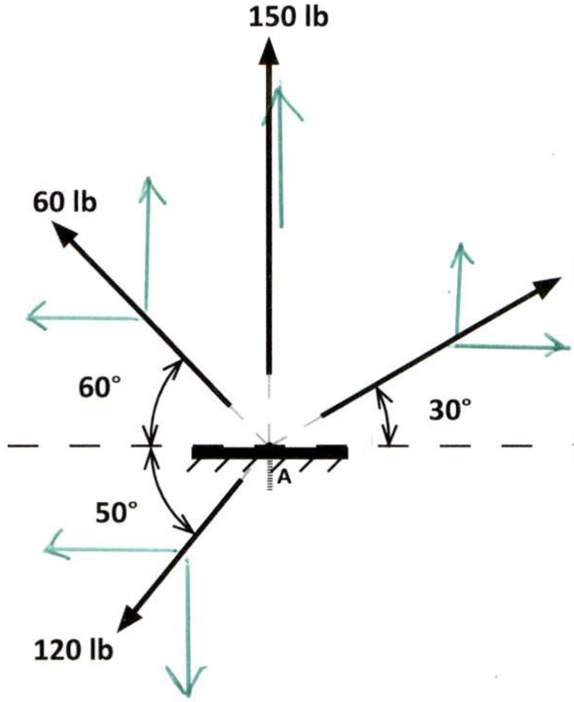
Θ is determined by which Quadrant the Resultant lies in:

Quadrant I	$\Theta = \alpha$
Quadrant II	$\Theta = 180° - \alpha$
Quadrant III	$\Theta = 180° + \alpha$
Quadrant IV	$\Theta = 360° - \alpha$

Find R and Θ



Example 2: Four forces act on bolt A as shown. Determine the resultant of the forces on the bolt.



all angles are to be measured ccw from the +x-axis.

Solution.

Force (lb)	Direction (Θ)	$F_x = F\cos\Theta$ (lb)	$F_y = F\sin\Theta$ (lb)
50	30°	43.3	25
150	90°	0	150
60	120°	−30	52
120	230°	−77	−92
		$\Sigma F_x =$ −64	$\Sigma F_y =$ 135

magnitude

$R_x = \Sigma F_x = -64\text{ lb} = 64\text{ lb} \leftarrow$

$R_y = \Sigma F_y = 135\text{ lb} = 135\text{ lb} \uparrow$

Resultant is in the Second Quadrant

$R = \sqrt{R_x^2 + R_y^2} = \sqrt{64\text{ lb}^2 + 135\text{ lb}^2} = 149\text{ lb}$ (magnitude)

Direction

$\alpha = \tan^{-1}\left|\frac{R_y}{R_x}\right| = \tan^{-1}\left|\frac{135}{64}\right| = 65°$

$\Theta = 180° - \alpha = 180° - 65° = 115°$

$R = 149\text{ lb} \measuredangle 115°$