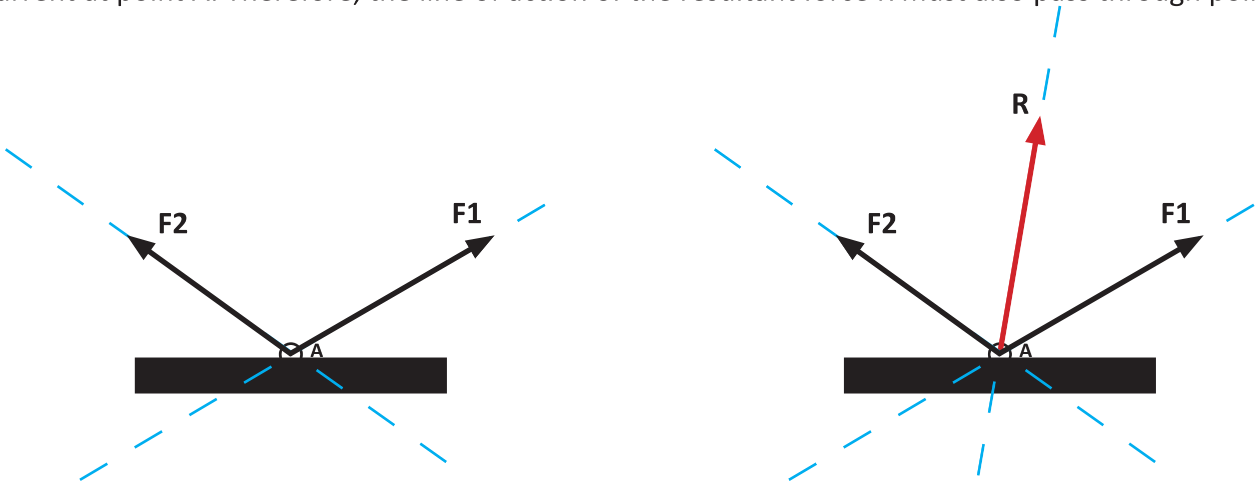
Chapter 2 - Resultant of Coplanar Force Systems
Reading: Chapter 2 - Pages 39-79

2-1
Introduction

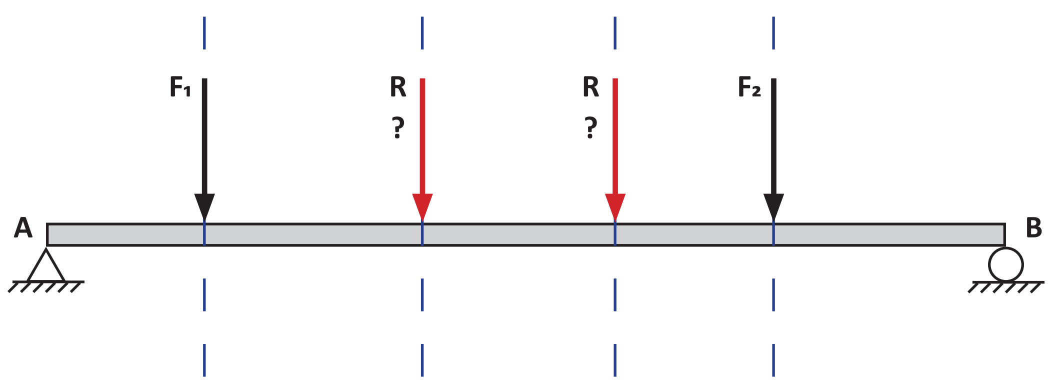
Two systems of forces are said to be equivalent if they produce the same mechanical effect on a rigid body. A single force that is equivalent to a given force system is called the resultant of the force system.

Resultant of a Concurrent Coplanar Force System
The forces F1 and F2 are acting on the rigid body at point A as shown. The line of action of each force are concurrent at point A. Therefore, the line of action of the resultant force R must also pass through point A.



<u>Methods for Determing the Resultant of a Concurrent Coplanar Force System</u>
Parallelogram Law
Triangle Rule
Graphically
Rectangular Components

Resultant of a Nonconcurrent Coplanar Force Systems
Any system of nonconcurrent coplanar forces can be replaced by a single resultant that is equivalent to the given force system.
The location of the line of action of the resultant is not immediately known.

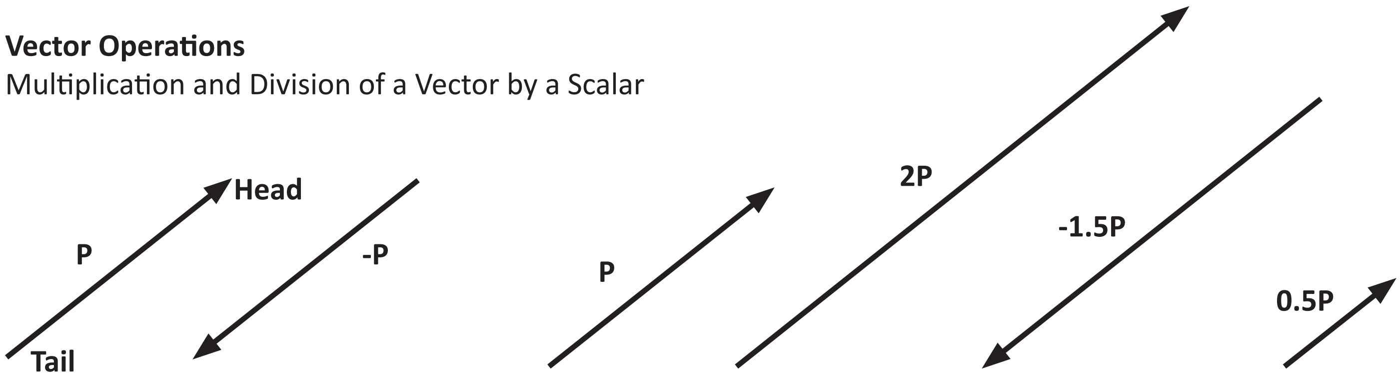


To determine the line of action of the resultant of a nonconcurrent coplanar force system, we will introduce the concepts of the moment of a force

2-2
Vector Representation

Scalars and Vectors
Definitions
Scalar: Any quantity possessing magnitude (size) only, such as mass, volume, temperature
Vector: Any quantity possessing both magnitude and direction, such as force, velocity, and momentum

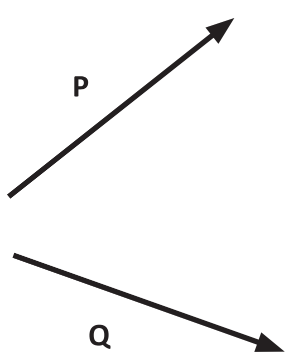
Vector Operations
Multiplication and Division of a Vector by a Scalar

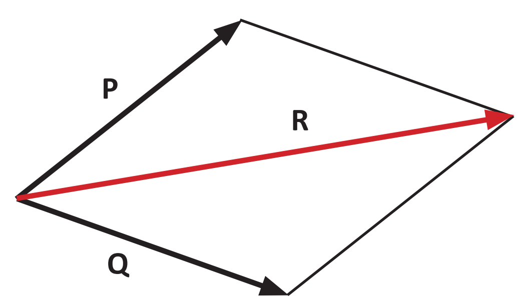


Vector Addition
Two vectors **P** and **Q** may be added to form a "resultant" vector **R = P + Q**
Vector addition is commutative: **P + Q** = **Q + P**

2-3
Resultant of Concurrent Forces

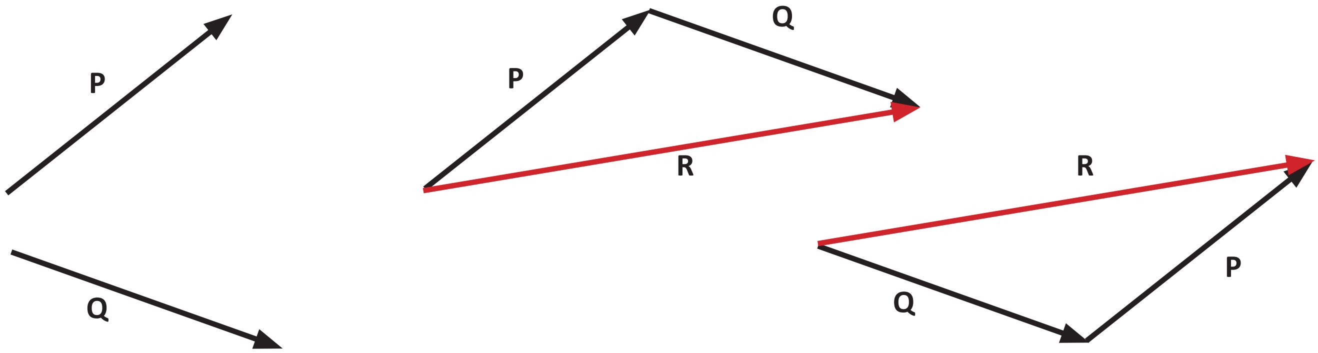
Method - Parallelogram Law
P and **Q** are joined at their tails.
Parallel lines drawn from the head of each vector intersect at a common point, forming the adjacent sides of a parallelogram.
Resultant **R** is the diagonal of the parallelogram, which extends from the tails of **P** and **Q** to the intersection of the lines.





Method - Triangle Rule

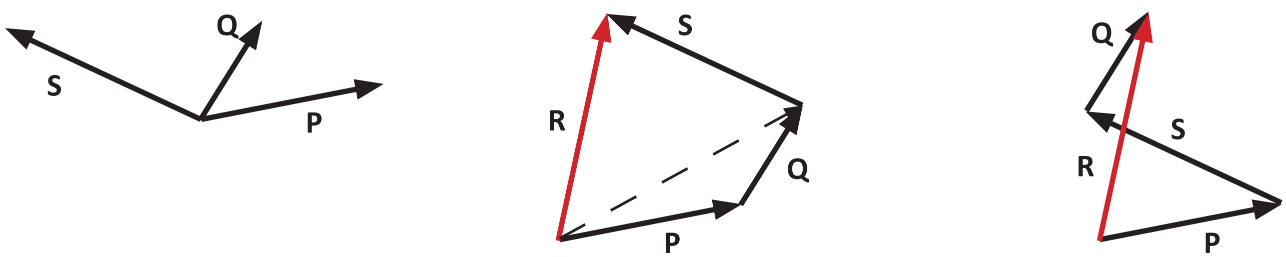
P and **Q** are added in a "head-to-tail" fashion by connecting the head of **P** to the tail of **Q**.
Resultant **R** extends from the tail of **P** to the head of **Q** forming a triangle.



Method - Polygon Rule

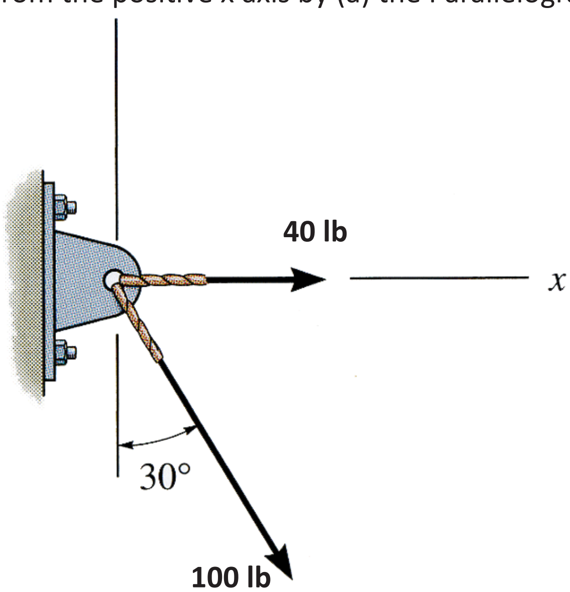
The sum of three or more concurrent coplanar vectors may be accomplished by adding two vectors successively.

Vectors **P**, **Q**, and **S** can be added successively by first finding **P** + **Q**, then adding **S** to **P** + **Q**.
Forms a polygon called a force polygon.



or,
Vectors **P**, **Q**, and **S** can be added successively by first finding **P** + **S**, then adding **Q** to **P** + **S**.
Forms a polygon called a force polygon.

Example 1: Determine the magnitude of the resultant force and its direction measured counter-clockwise from the positive x axis by (a) the Parallelogram Law and (b) the Triangle Rule.



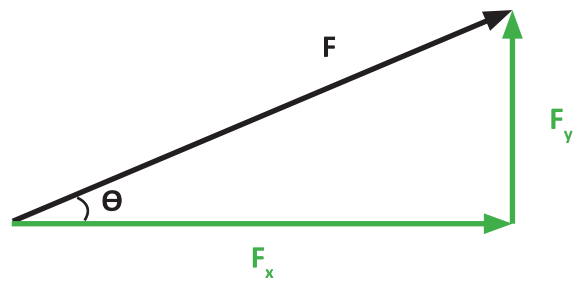
Parallelogram Law

Triangle Rule

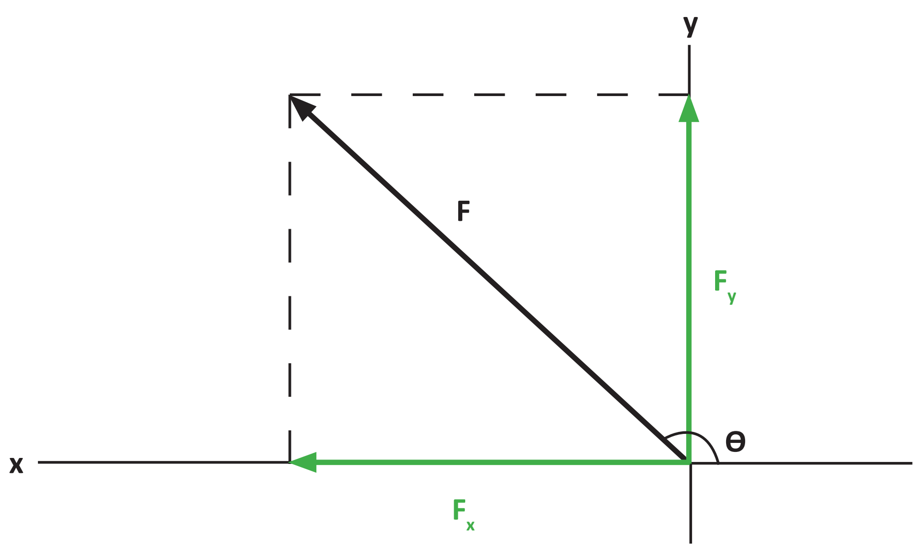
2-4
Rectangular Components

A vector can always be specified by its magnitude and direction, but it is often necessary or convenient to specify one by its rectangular components F_x and F_y (and F_z if the vector is three dimensional).

F_x, the x component of the vector F, is the x coordinate of the head of F when the vector F is drawn with its tail at the origin of a coordinate system. The y component of F, F_y, is similarly defined. F_x and F_y are illustrated in the figure below. Note that the rectangular components of a vector are real numbers that may be positive, zero, or negative.



The vector F shown above is a vector in the first quadrant.
The direction (angle Θ) of a vector in the first quadrant is between 0° and 90°

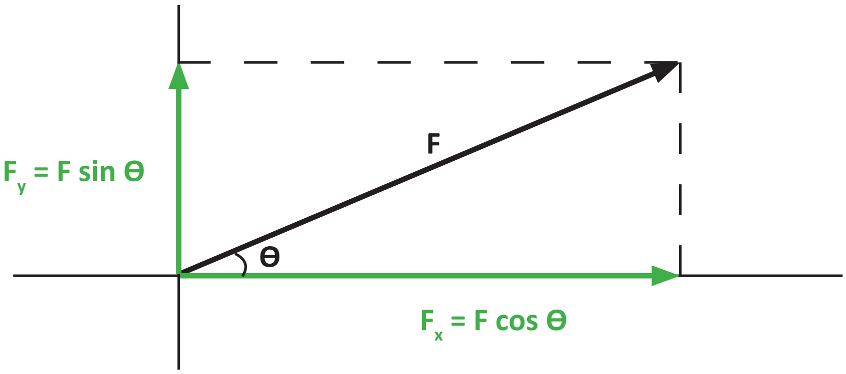


The vector F shown in the second diagram is a vector in the second quadrant.
The direction (angle Θ) of a vector in the second quadrant is between >90° and 180°

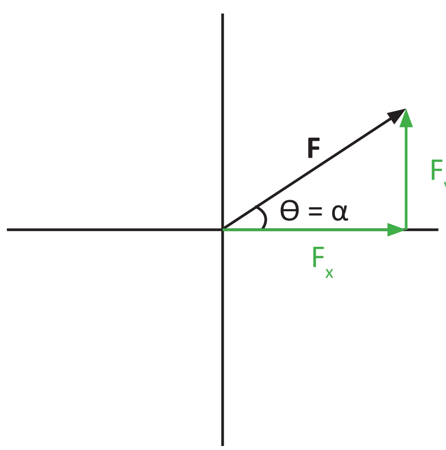
In the second quadrant x is negative and y is positive.

The **rectangular components** of vector F can be drawn beginning at the the tail of the vector. The x-component is drawn in the negative x direction (arrow to the left) and the y-component is drawn in the positive y direction (arrow upward). The dashed lines complete the "rectangle".

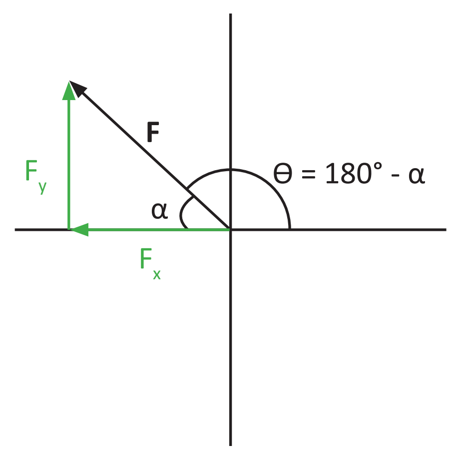
Any force can be resolved into its rectangular components Fx and Fy, which lie along the x and y axes respectively.



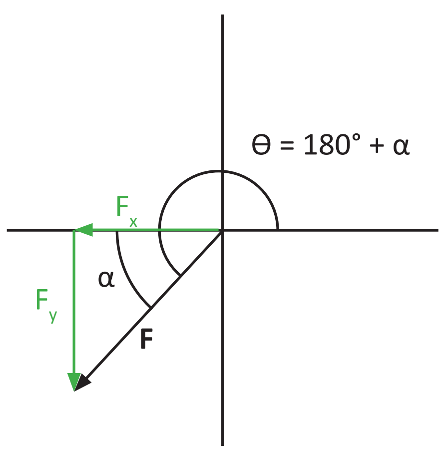
Magnitude	**Direction**	
$F = \sqrt{F_x^2 + F_y^2}$	$\Theta = \alpha$	(First Quadrant)
	$\alpha = \tan^{-1}\left\lvert\frac{F_y}{F_x}\right\rvert$	(see below)



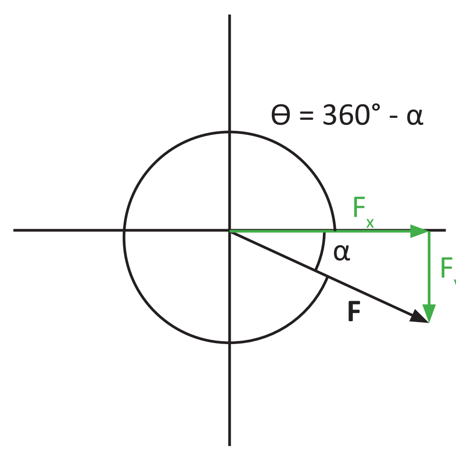
First Quadrant $\Theta = \alpha$



Second Quadrant $\Theta = 180° - \alpha$



Third Quadrant $\Theta = 180° + \alpha$



Fourth Quadrant $\Theta = 360° - \alpha$

2-5
Resultants by Rectangular Components

Using rectangular components, obtained by resolving forces in the system, the resultant can be obtained. The resultant force, just like any other force, can be thought of as being able to be resolved into x and y components R_x and R_y.

The magnitude of the components R_x and R_y is simply the sum of the x and y components of the forces in the system.

Magnitude

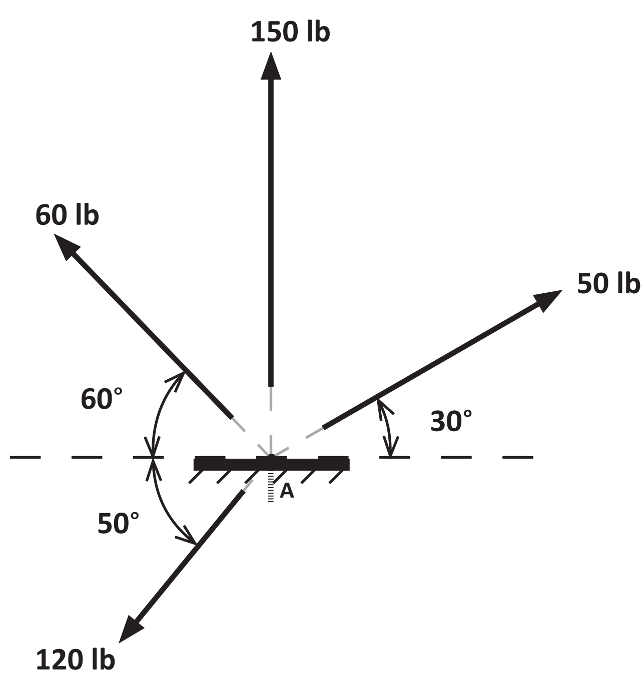
$$\left.\begin{array}{l} R_x = \sum F_x \\ R_y = \sum F_y \end{array}\right\} R = \sqrt{R_x^{\,2} + R_y^{\,2}}$$

Direction

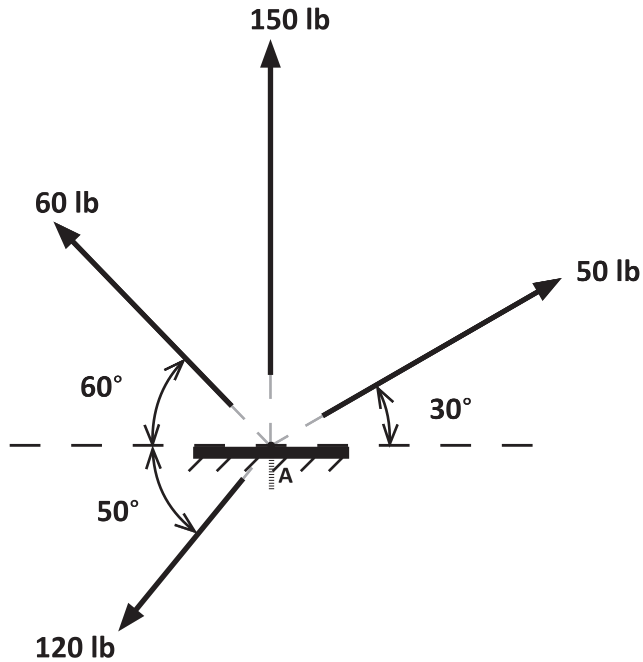
$$\alpha = \tan^{-1}\left|\frac{R_y}{R_x}\right|$$

Θ is determined by which Quadrant the Resultant lies in:

Quadrant I	Θ = α
Quadrant II	Θ = 180° - α
Quadrant III	Θ = 180° + α
Quadrant IV	Θ = 360° - α



Example 2: Four forces act on bolt A as shown. Determine the resultant of the forces on the bolt.



Solution.

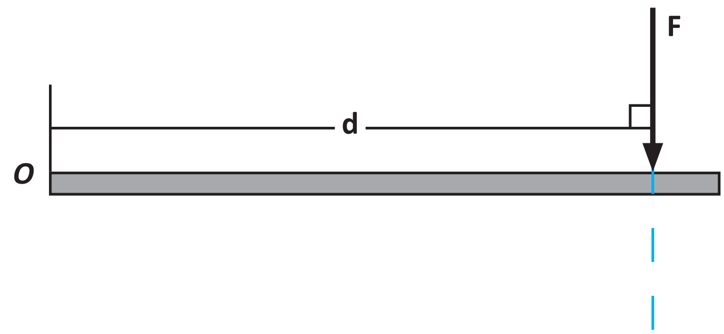
Force (lb)	**Direction (Θ)**	$F_x = F \cos \Theta$ **(lb)**	$F_y = F \sin \Theta$ **(lb)**
		$\sum F_x =$	$\sum F_y =$

2-6
Moment of a Force

Force
Tends to move a body along the line of action of the force.
If the body is constrained there may also be a tendency to **rotate** the body about a point or axis.

The **moment** of a force about a point or axis provides a measure of the tendency of the force to cause the body to rotate about the point or axis.

Consider the force **F** and point *O*.



The moment M_o about point *O* (moment center) is equal to the magnitude of the force **F** multiplied by the perpendicular distance d (moment arm) from *O* to the line of action of the force.

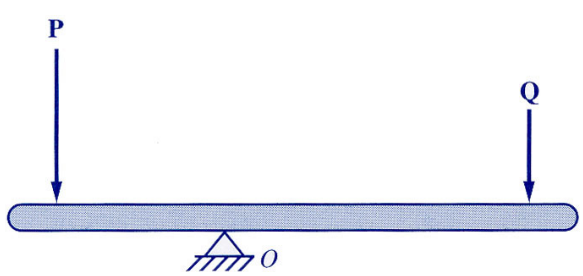
The **magnitude of moment** is given by the equation: $M_o = F\ d$

Units

U.S. Customary units:	lb-ft or lb-in
SI units:	N-m or kN-m

Direction of Moments
CCW is considered +M 

CW is considered -M 

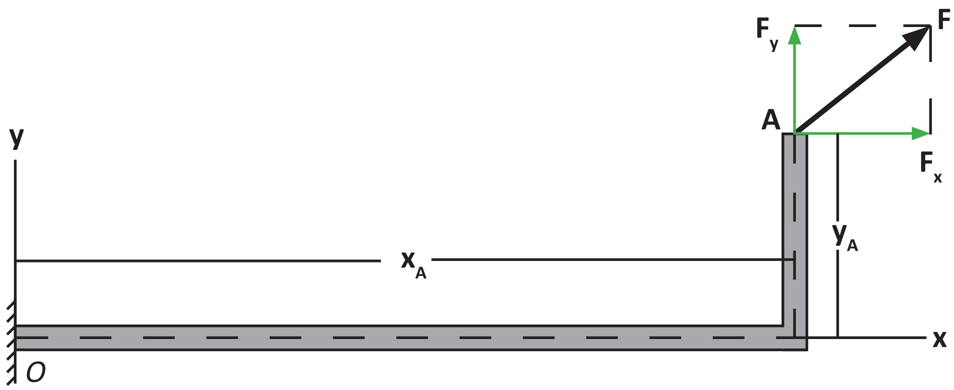


The Force P causes a counterclockwise (CCW) rotation 

The Force Q causes clockwise (CW) rotation 

Direction of moments MUST be specified.

Varignon's Theorem
The moment caused by the resultant force (of some system of forces) about some arbitrary point is equal to the sum of the moments due to all of the component forces of the system.

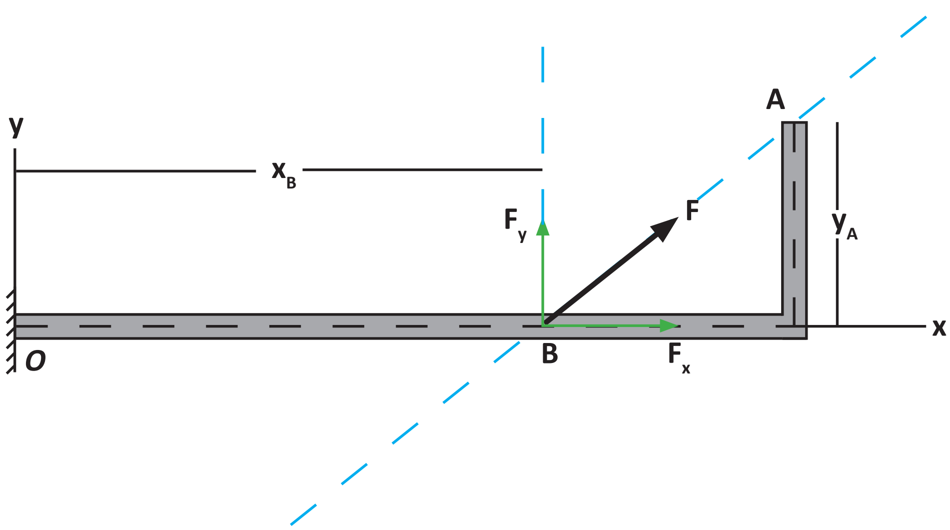


CCW   CW

$$M_O = +F_y\ x_A + -F_x y_A$$

Principle of Transmissibility
Moment arm is independent of the point of application of a force.

As long as the magnitude, direction, and the line of action of a force are defined, the moment of the force about a given point may be determined by placing the force at any point along its line of action.

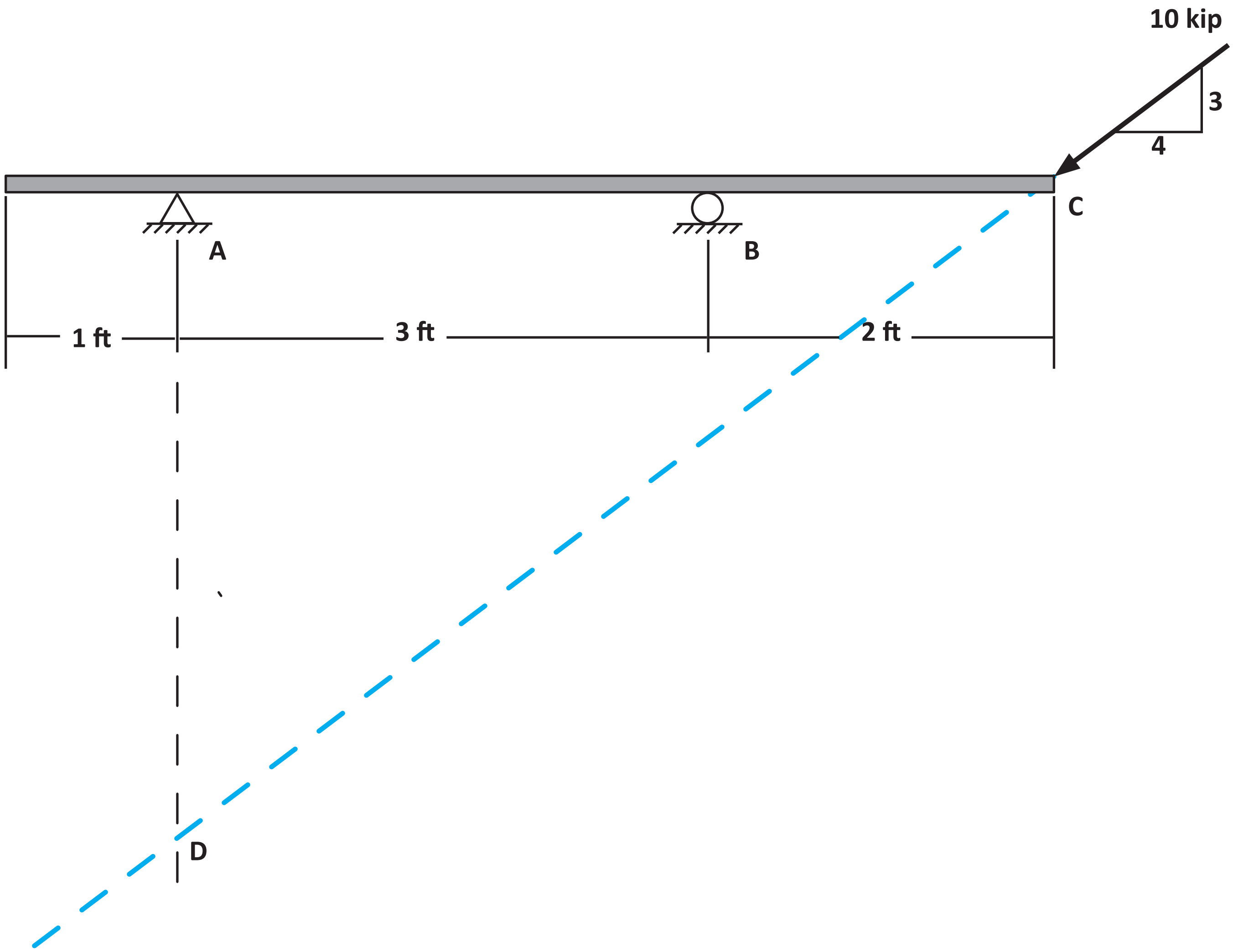


CCW

$$M_O = +F_y\ x_B + F_x \times 0 = F_y\ x_B$$

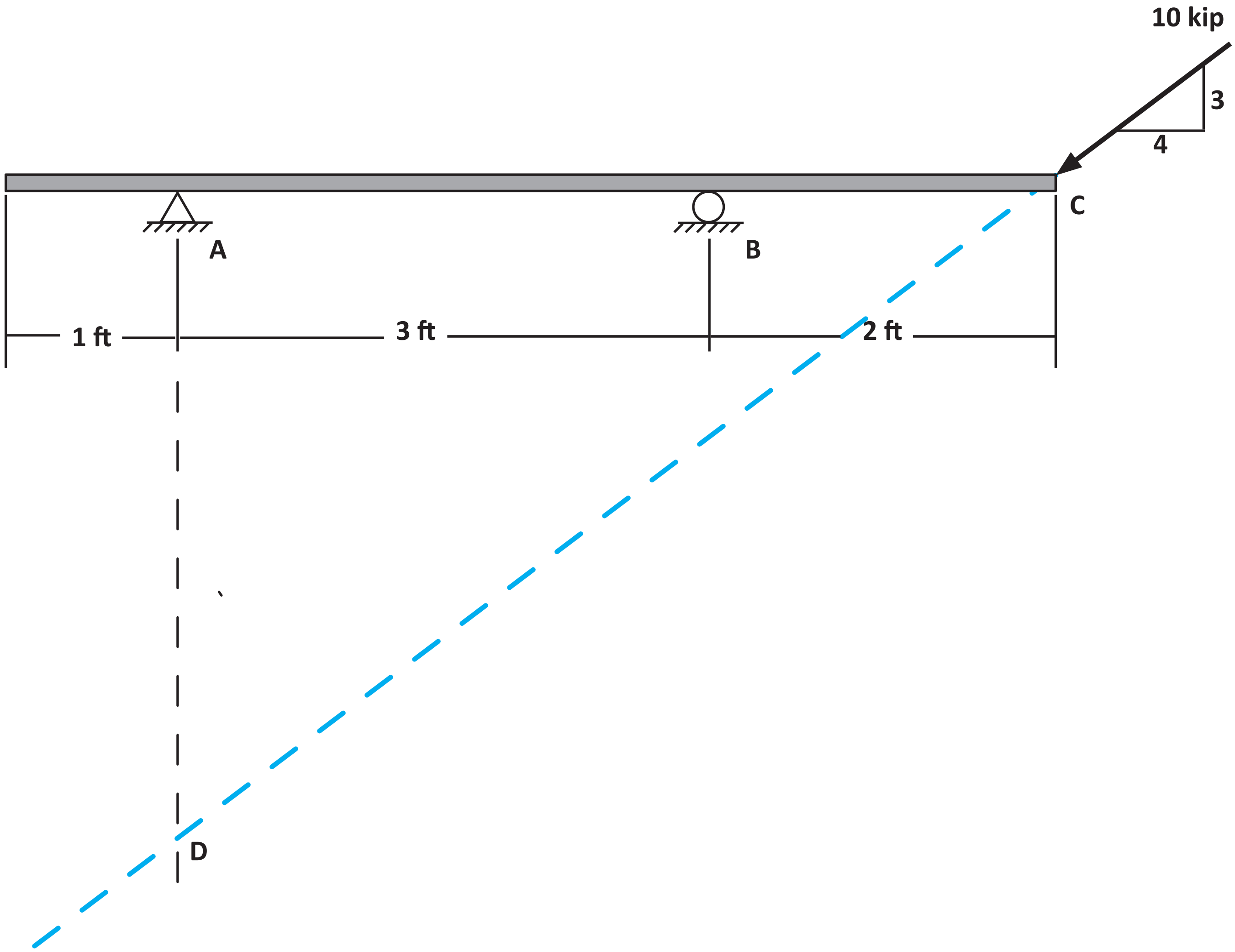
Since the line of action of the component F_x passes through the moment center *O*, it produces no moment about *O*.

Example 3: Determine the moment about A of the 10 kip force shown by (a) using the definition directly, (b) resolving the force into horizontal and vertical components at C and use Varignon's Theorem, and (c) resolving the force into horizontal and vertical components at D using the principle of transmissibilty and Varignon's Theorem.

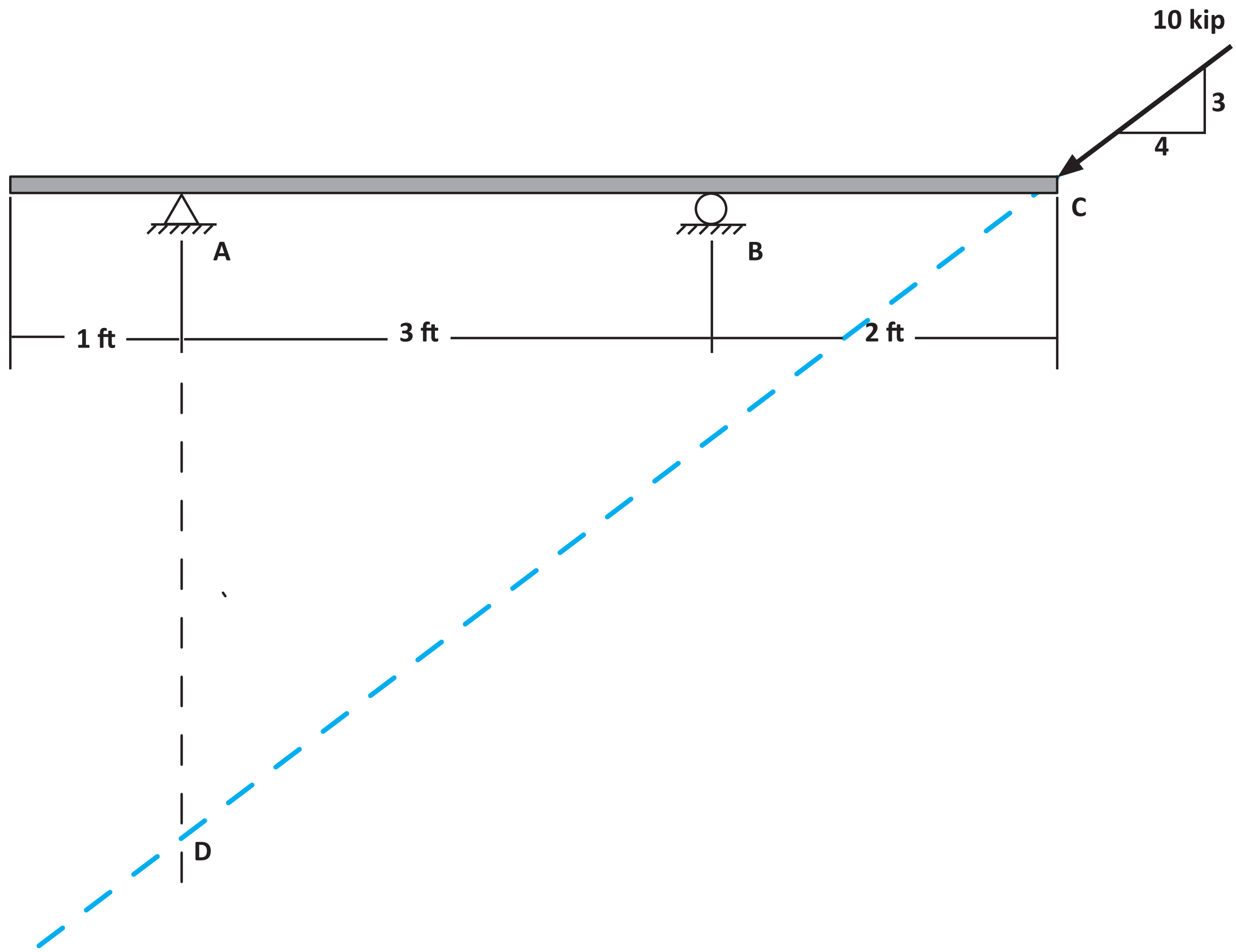


Solution.
(a) Using the definition directly

(b) resolving the force into horizontal and vertical components at C and use Varignon's Theorem

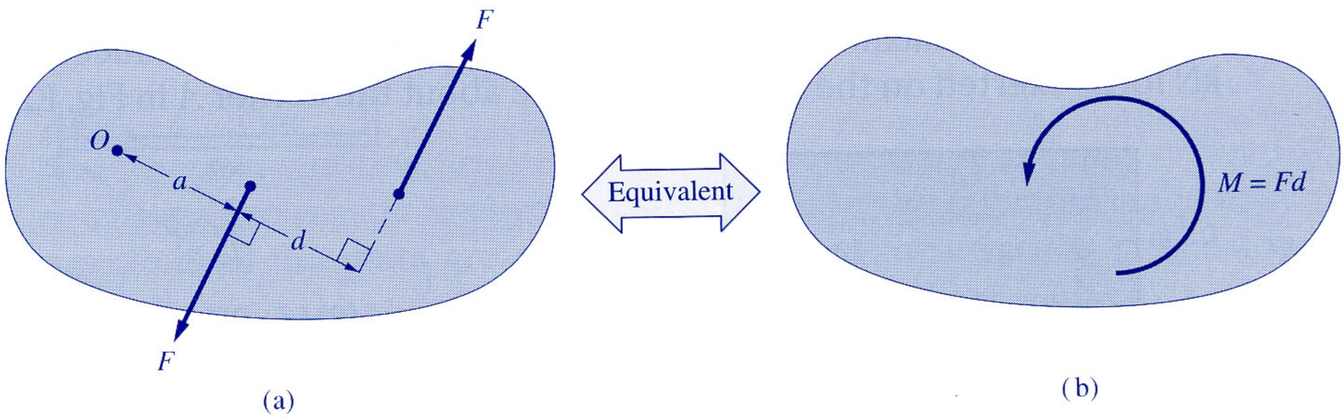


(b) resolving the force into horizontal and vertical components at D using the principle of transmissibilty and Varignon's Theorem.



2-8
Couple

Couple
A couple is defined as two parallel forces that have the same magnitude, have opposite directions, and are separated by a perpendicular distance d.



Since the resultant force is zero, the only effect of a couple is to produce a rotation or tendency of rotation in a specified direction.
The moment produced by a couple is called a couple moment.

Moment of a Couple
The moment of a couple about an arbitrary point *O* is: $M = F(a + d) - Fa = Fd$

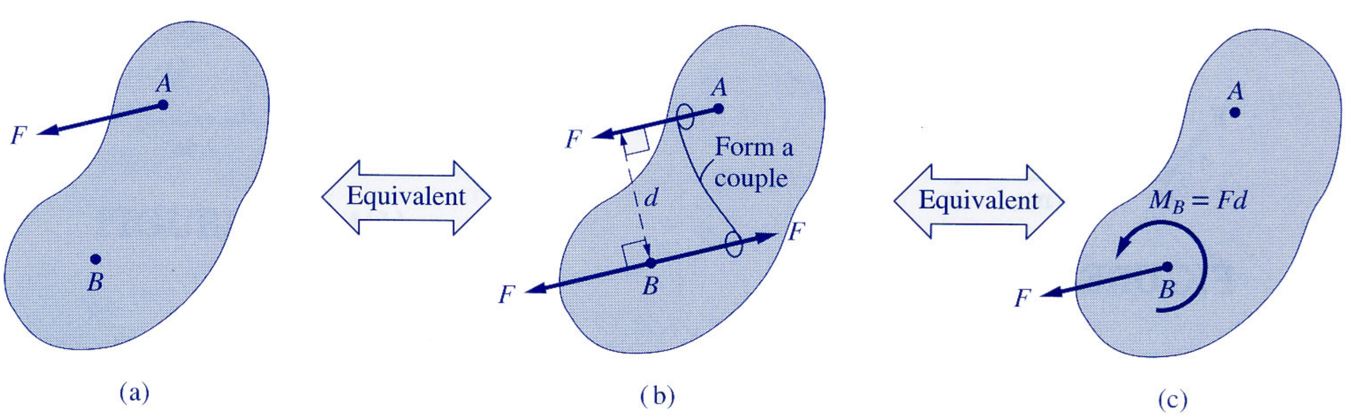
$M = Fd$

Since *O* is an arbitrary point, the moment of a couple about any point is equal to the magnitude of the forces times the perpendicular distance between the forces.
A tire iron is an excellent example of a couple in action.

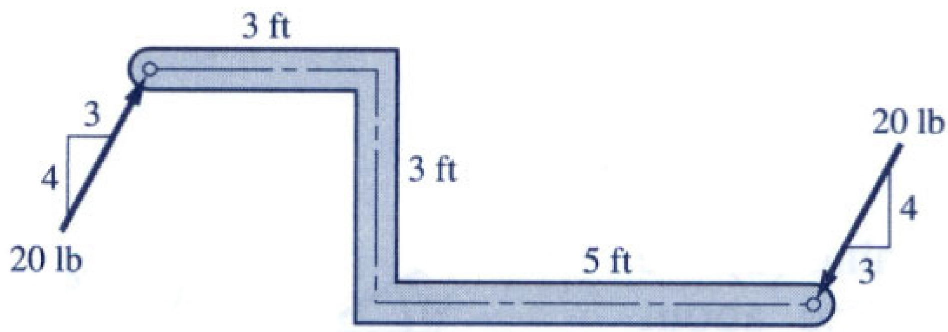
2-9
Replacing a Force with a Force-Couple System

Force systems are said to be equivalent if they produce the same mechanical effect on a rigid body.
Equivalent system of forces: same resultant force and same resultant moment about the same point.

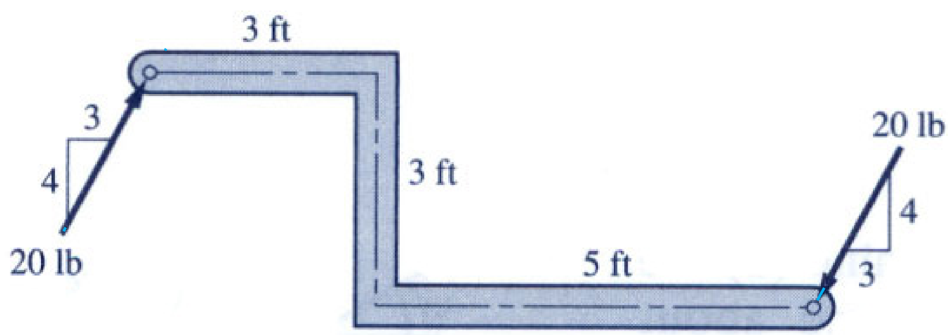
A force acting on a rigid body may be replaced by an equivalent force-couple system at an arbitrary point *O* consisting of the force applied at *O* and a couple having a moment equal to the moment about *O* of the given force at the original location.



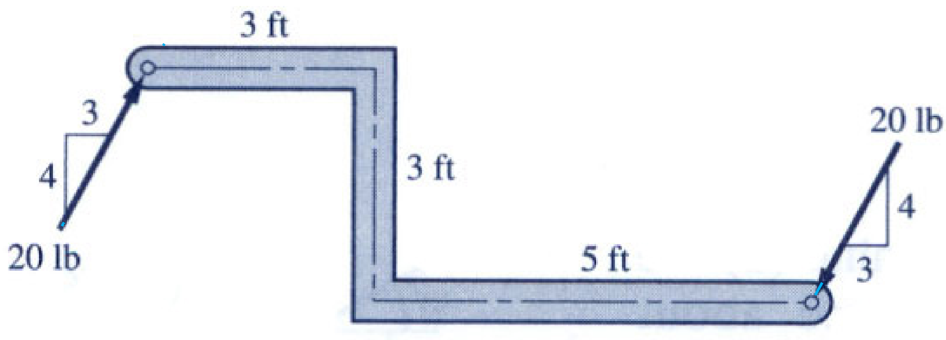
Example 4: Determine the moment of the couple acting on the body shown.



Solution.
Use Varignon's Theorem



Solution.
Use the definition directly



2–51 Replace the 600-lb force acting on the connection in Fig. P2–51 with an equivalent force-couple system at the center of rivet *B*.

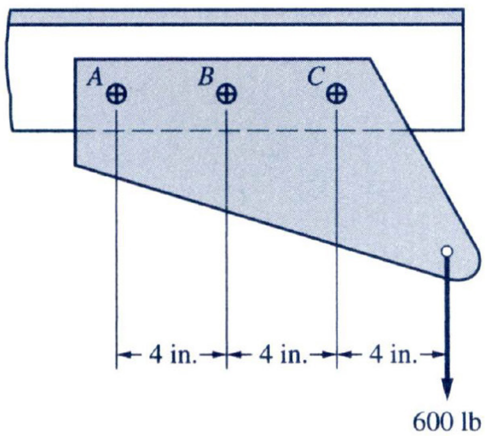


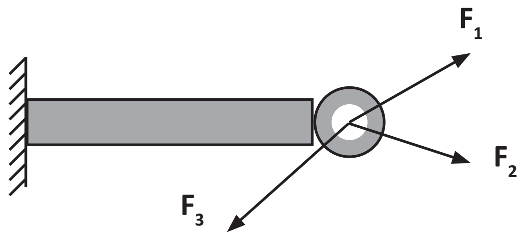
FIGURE P2–51

2-10

Resultant of a Nonconcurrent Coplanar Force system

Concurrent Coplanar Force System

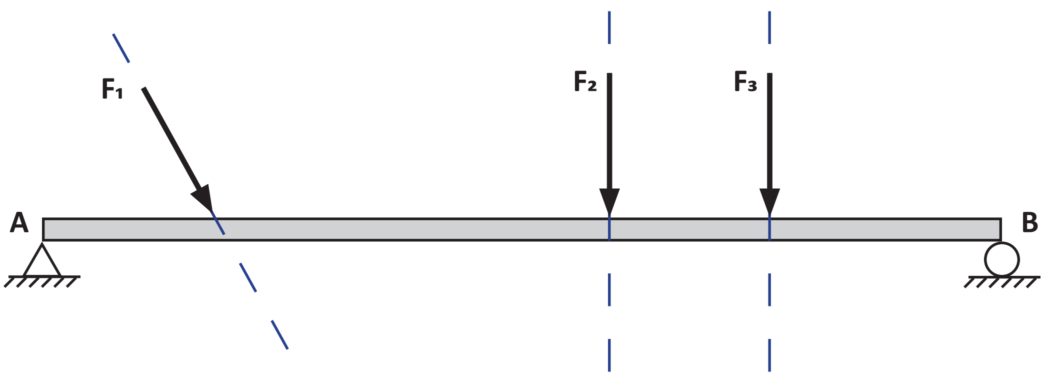
The line of action of the Resultant passes through a common point.



Nonconcurrent Coplanar Force System

There is no point of concurrency.

The **location** of the line of action of the resultant (R) is not immediately known.



Magnitude of the Resultant

$R_x = \sum F_x$

$R_y = \sum F_y$

$R = \sqrt{R_x^{\,2} + R_y^{\,2}}$

Direction of the Resultant

$$\alpha = \tan^{-1}\left|\frac{R_y}{R_x}\right|$$

Θ is determined by which Quadrant the Resultant lies in.

Location of the Resultant

The location of the line of action of the resultant can be determined by the requirement of the moments.

If two force systems are equivalent, the resultant moments of the two systems about an arbitrary point must be equal.

$$R_y\,\bar{x} = \sum M_A$$

2–64 Find the magnitude, direction, and location of the resultant of the three forces acting on the beam in Fig. P2–64.

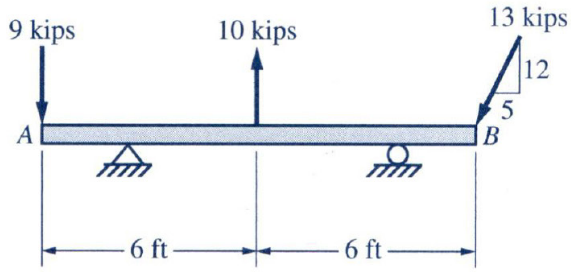


FIGURE P2–64

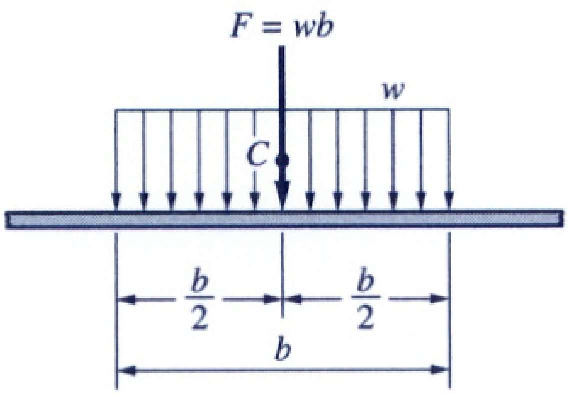
Solution.

2-11
Resultant of Distributed Line Loads

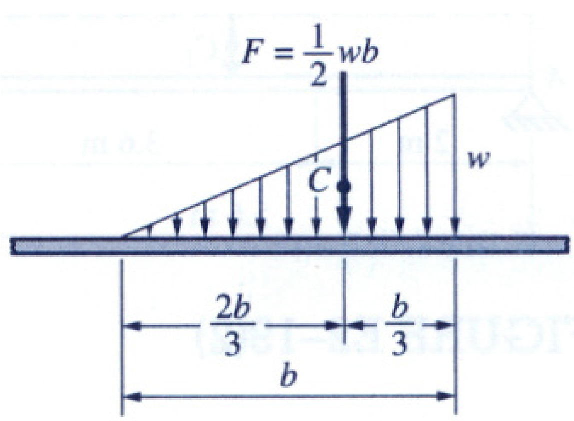
Distributed Load
A distributed load occurs whenever the load applied to a body is not concentrated at a point. A distributed load could be exerted along a line, over an area, or throughout an entire solid body.

Load Intensity
A distributed load along a line is characterized by a load intensity expressed as force per unit length.
Units:
U.S. lb/ft
S.I. N/m or kN/m

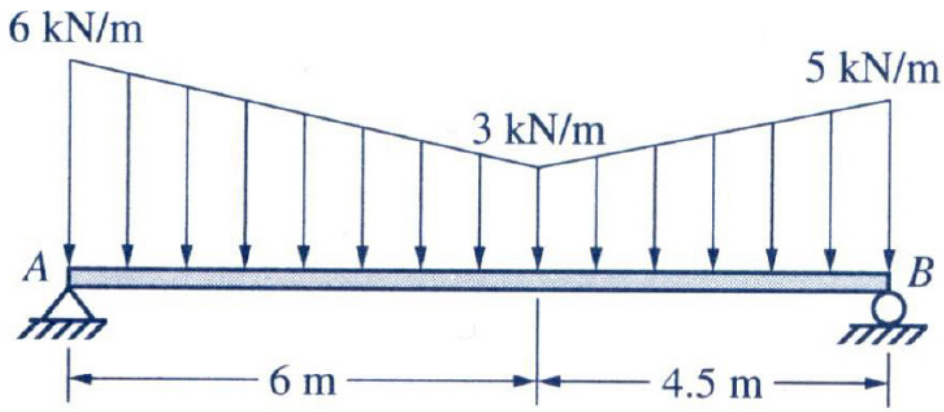
Uniform Load
A distributed load with constant load intensity w is called a uniform load.



Triangular Load
A triangular load is a distributed load whose intensity varies linearly from zero to a maximum intensity w.



Example 7: Replace the loading on the beam with an equivalent resultant force and specify its location with respect to point A.



Solution.