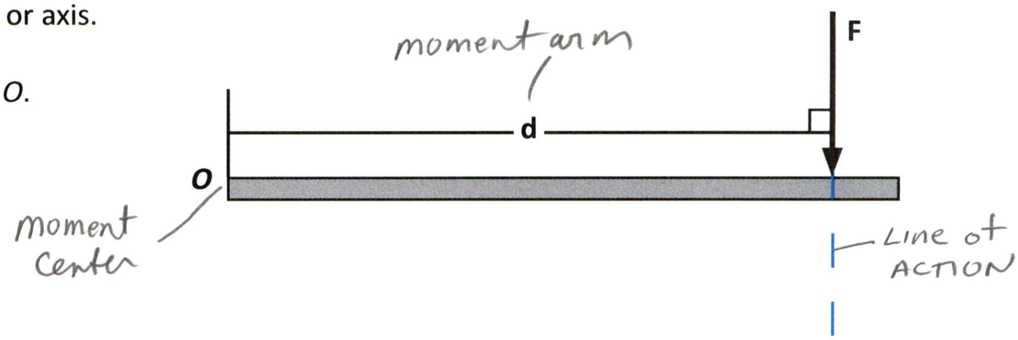
2-6
Moment of a Force

Force

Tends to move a body along the line of action of the force.
If the body is constrained there may also be a tendency to **rotate** the body about a point or axis.

The **moment** of a force about a point or axis provides a measure of the tendency of the force to cause the body to rotate about the point or axis.

Consider the force **F** and point *O*.



The moment M_o about point *O* (moment center) is equal to the magnitude of the force **F** multiplied by the perpendicular distance d (moment arm) from *O* to the line of action of the force.

The **magnitude of moment** is given by the equation: $M_o = F\,d$

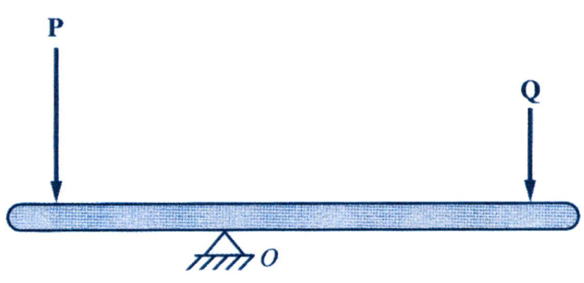
Units

U.S. Customary units:	lb-ft or lb-in
SI units:	N-m or kN-m

Direction of Moments

CCW is considered +M

CW is considered -M



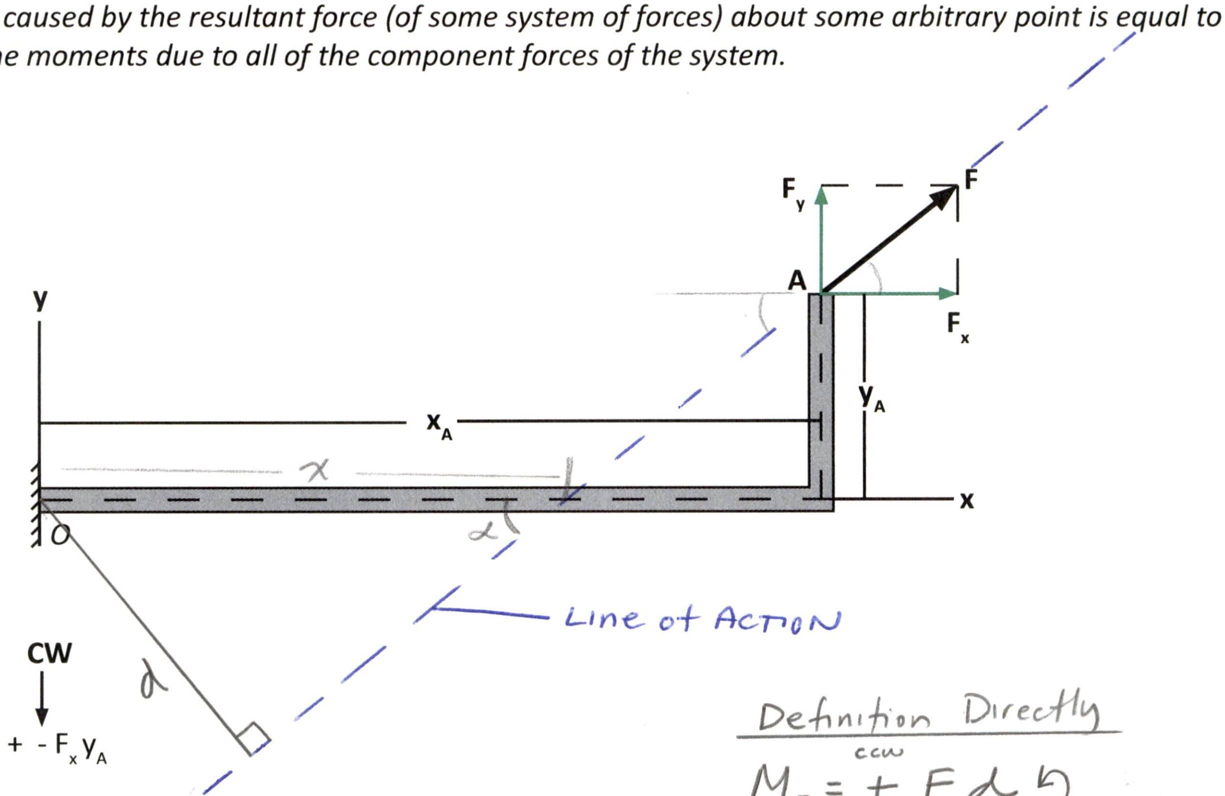
The Force P causes a counterclockwise (CCW) rotation +M

The Force Q causes clockwise (CW) rotation −M

Direction of moments MUST be specified.

Varignon's Theorem

The moment caused by the resultant force (of some system of forces) about some arbitrary point is equal to the sum of the moments due to all of the component forces of the system.



CCW CW

$M_O = +F_y\, x_A + -F_x\, y_A$

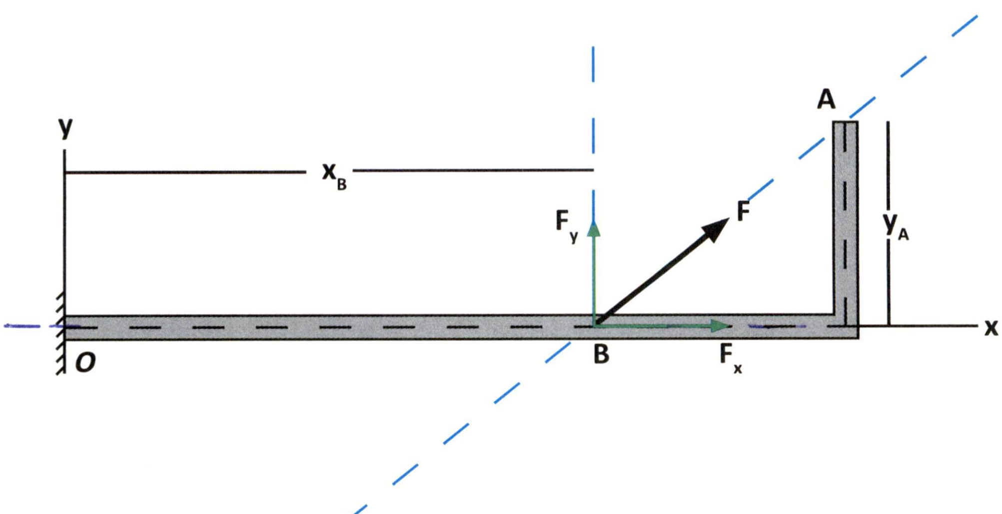
Definition Directly

$M_O \overset{ccw}{=} +F d$

Principle of Transmissibility

Moment arm is independent of the point of application of a force.

As long as the magnitude, direction, and the line of action of a force are defined, the moment of the force about a given point may be determined by placing the force at any point along its line of action.

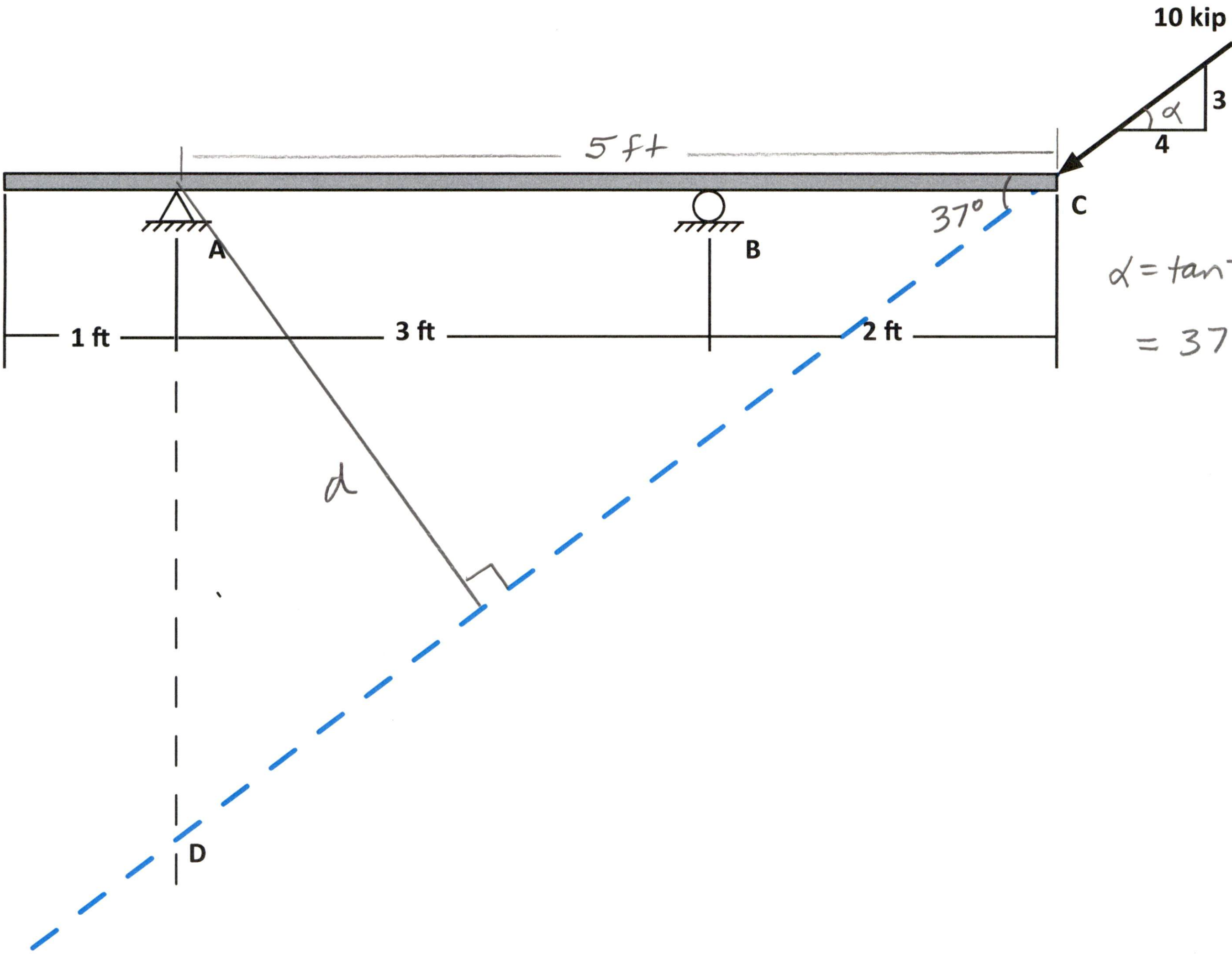


CCW

$M_O = +F_y\, x_B + F_x \times 0 = F_y\, x_B$

Since the line of action of the component F_x passes through the moment center O, it produces no moment about O.

Example 3: Determine the moment about A of the 10 kip force shown by (a) using the definition directly, (b) resolving the force into horizontal and vertical components at C and use Varignon's Theorem, and (c) resolving the force into horizontal and vertical components at D using the principle of transmissibilty and Varignon's Theorem.



$\alpha = \tan^{-1}\frac{3}{4}$

$= 37°$

Solution.

(a) Using the definition directly

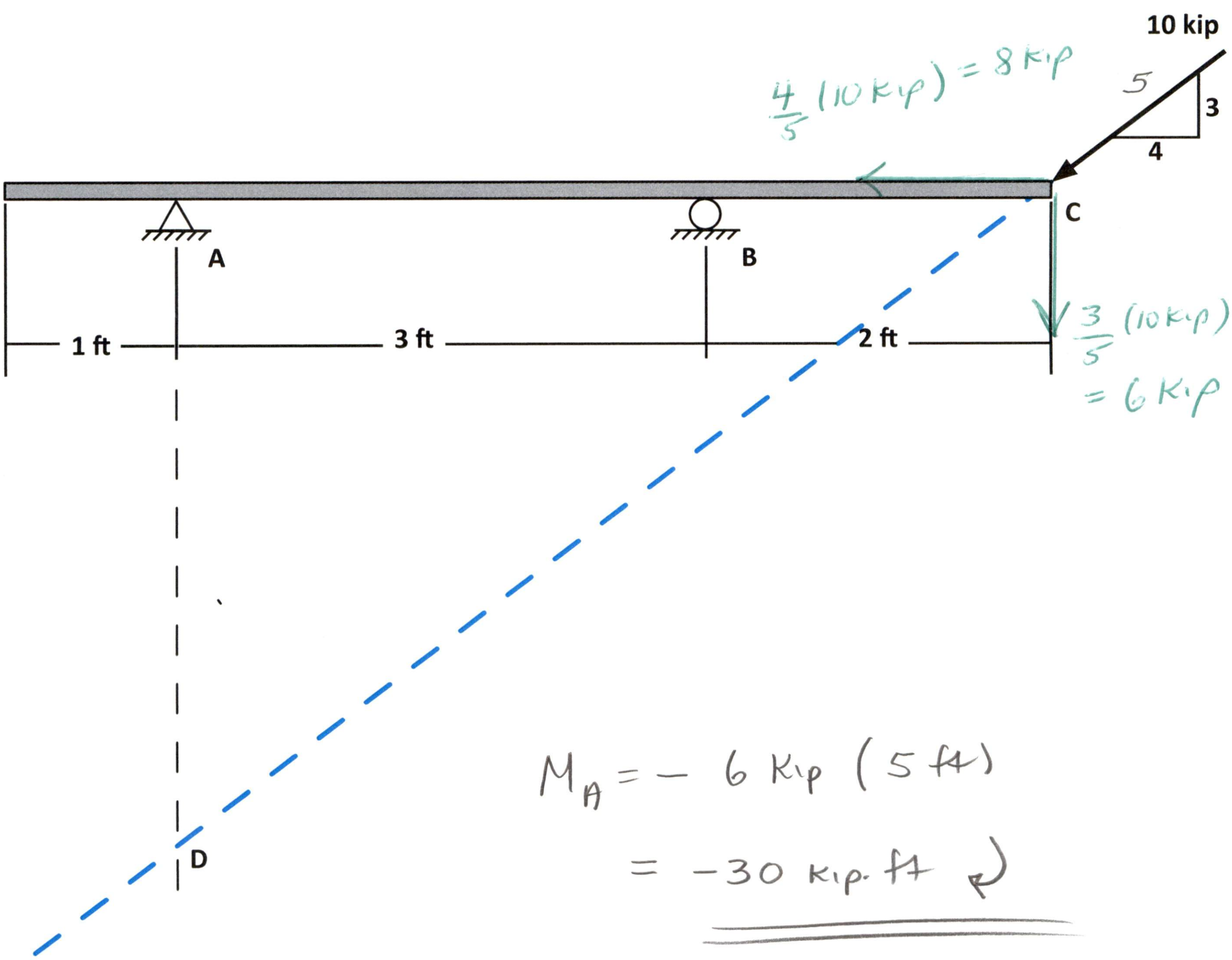
Find the Moment arm d

$$\sin 37° = \frac{d}{5ft}$$

$$d = 5ft \sin 37° = 3 ft$$

$$M_A = -10 kip\,(3ft) = \underline{-30 \text{ kip}\cdot\text{ft}}$$

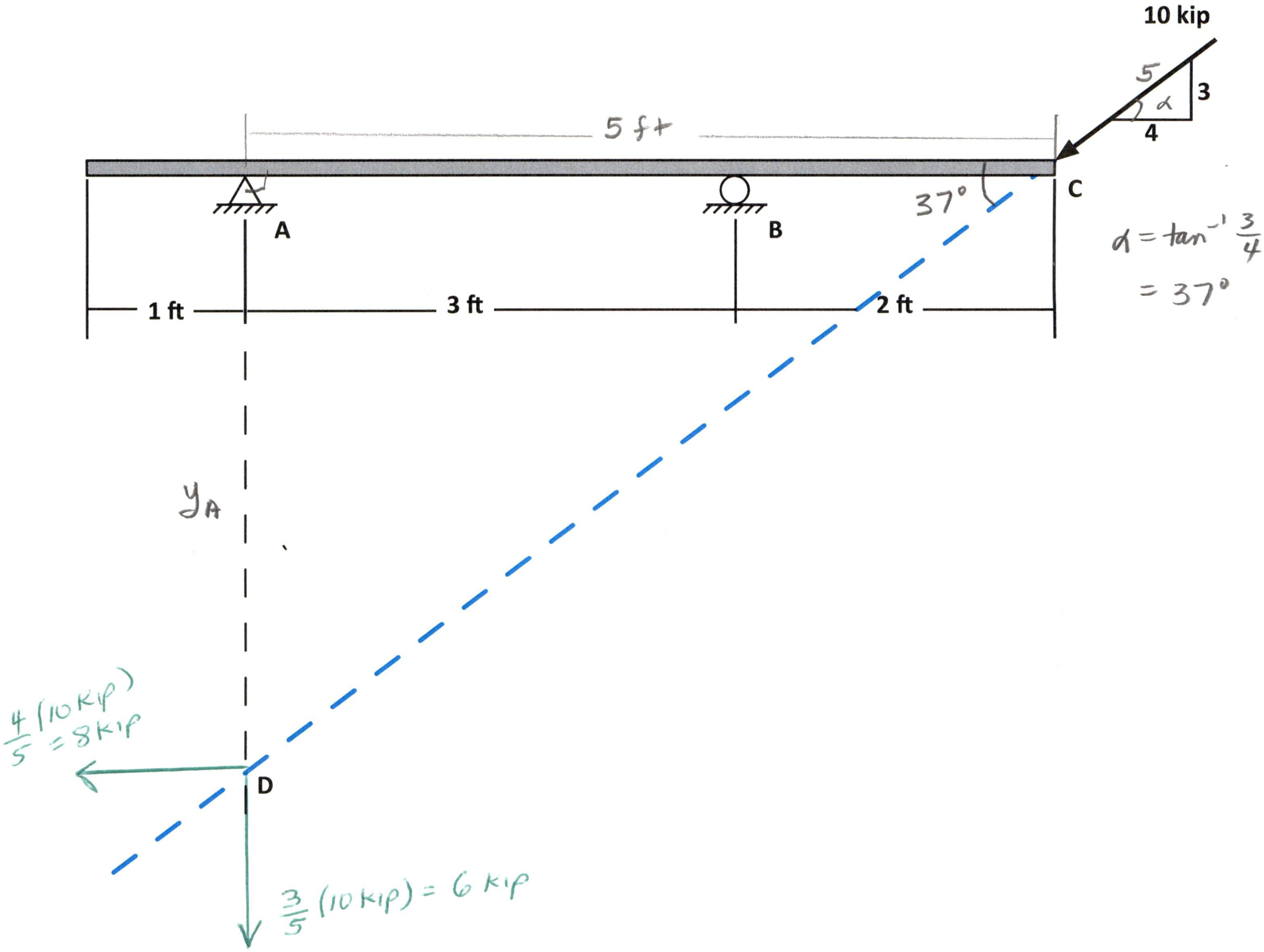
(b) resolving the force into horizontal and vertical components at C and use Varignon's Theorem



$$M_A = -6\ kip\ (5\ ft)$$

$$= \underline{\underline{-30\ kip \cdot ft}}$$

(b) resolving the force into horizontal and vertical components at D using the principle of transmissibilty and Varignon's Theorem.



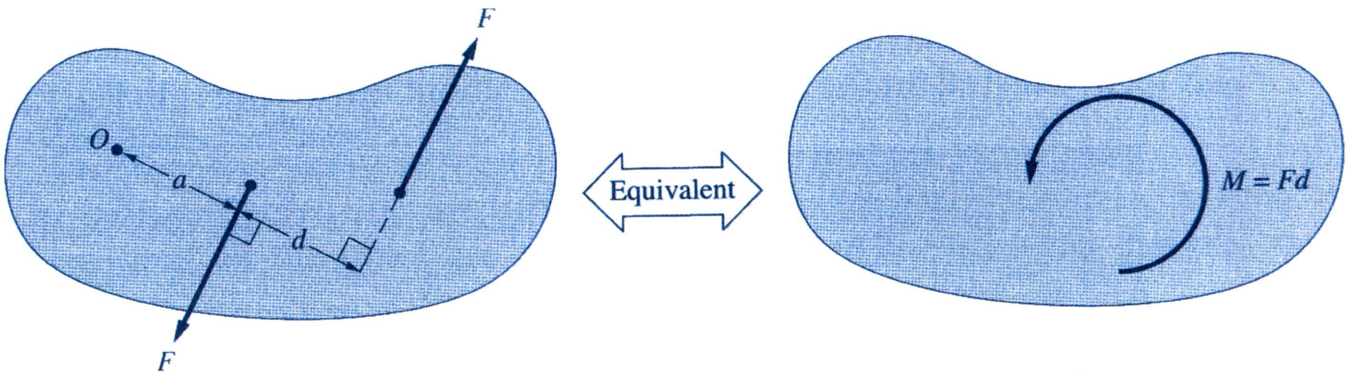
$$\tan 37° = \frac{y_A}{5\,ft}$$

$$y_A = 5\,ft\,(\tan 37°) = 3.8\,ft$$

$$M_A = -8\,kip\,(3.8\,ft) = \underline{-30\,kip \cdot ft}$$

2-8
Couple

Couple
A couple is defined as two parallel forces that have the same magnitude, have opposite directions, and are separated by a perpendicular distance d.



(a) (b)

Since the resultant force is zero, the only effect of a couple is to produce a rotation or tendency of rotation in a specified direction.
The moment produced by a couple is called a couple moment.

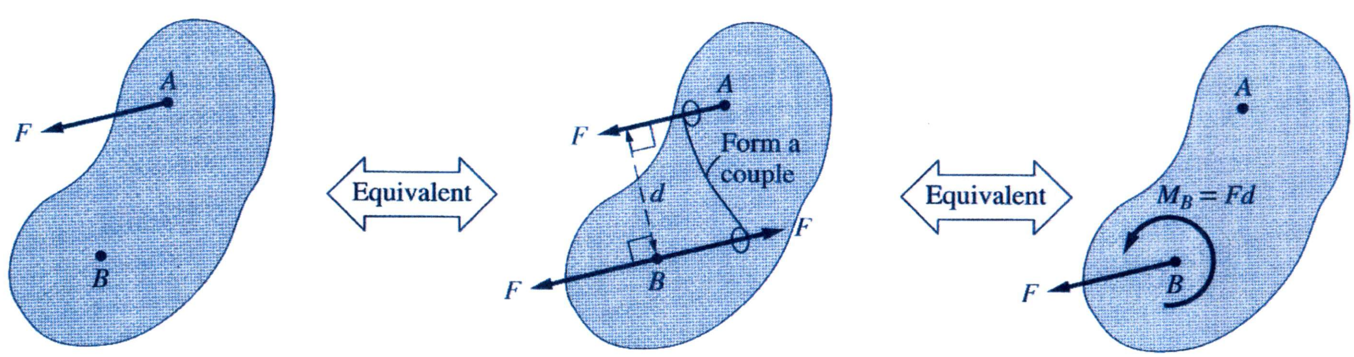
Moment of a Couple
The moment of a couple about an arbitrary point *O* is: $M_O = F(a + d) - Fa = Fd$
$M_{\text{¢}} = Fd$

Since *O* is an arbitrary point, the moment of a couple about any point is equal to the magnitude of the forces times the perpendicular distance between the forces.
A tire iron is an excellent example of a couple in action.

2-9
Replacing a Force with a Force-Couple System

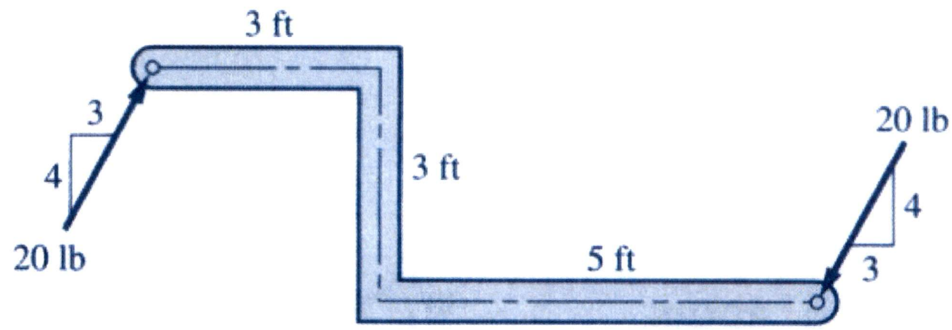
Force systems are said to be equivalent if they produce the same mechanical effect on a rigid body.
Equivalent system of forces: same resultant force and same resultant moment about the same point.

A force acting on a rigid body may be replaced by an equivalent force-couple system at an arbitrary point *O* consisting of the force applied at *O* and a couple having a moment equal to the moment about *O* of the given force at the original location.

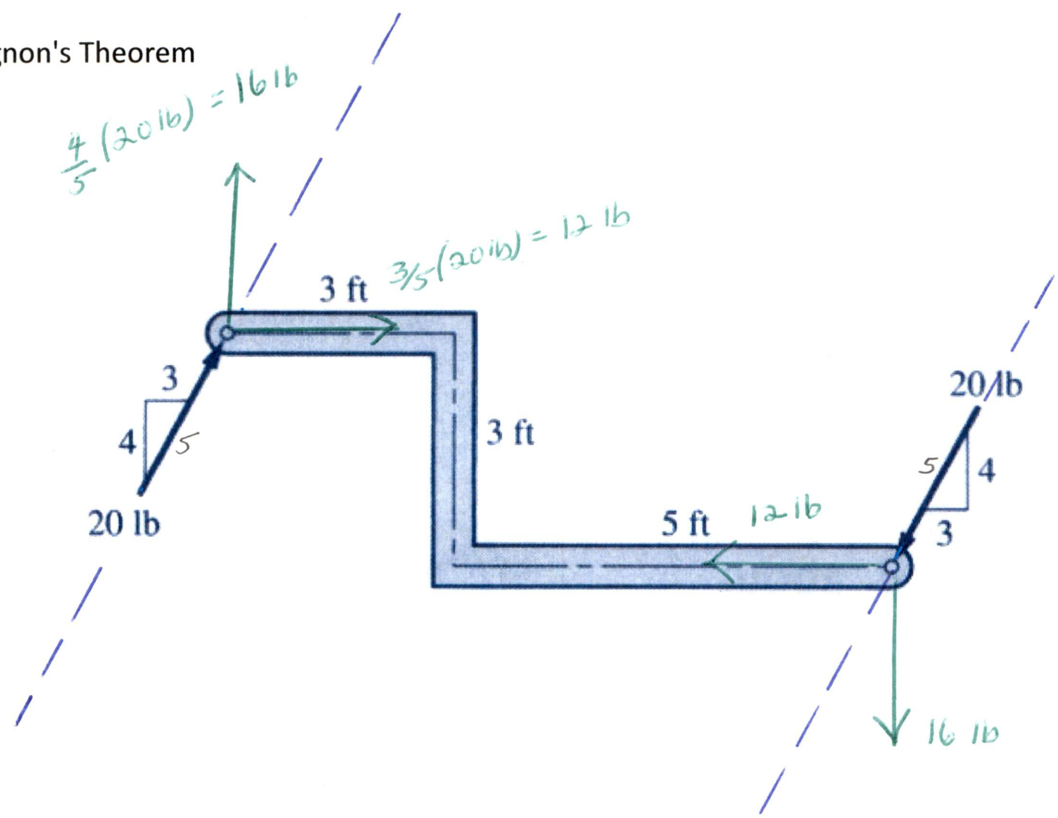


(a) (b) (c)

Example 4: Determine the moment of the couple acting on the body shown.



Solution.
Use Varignon's Theorem

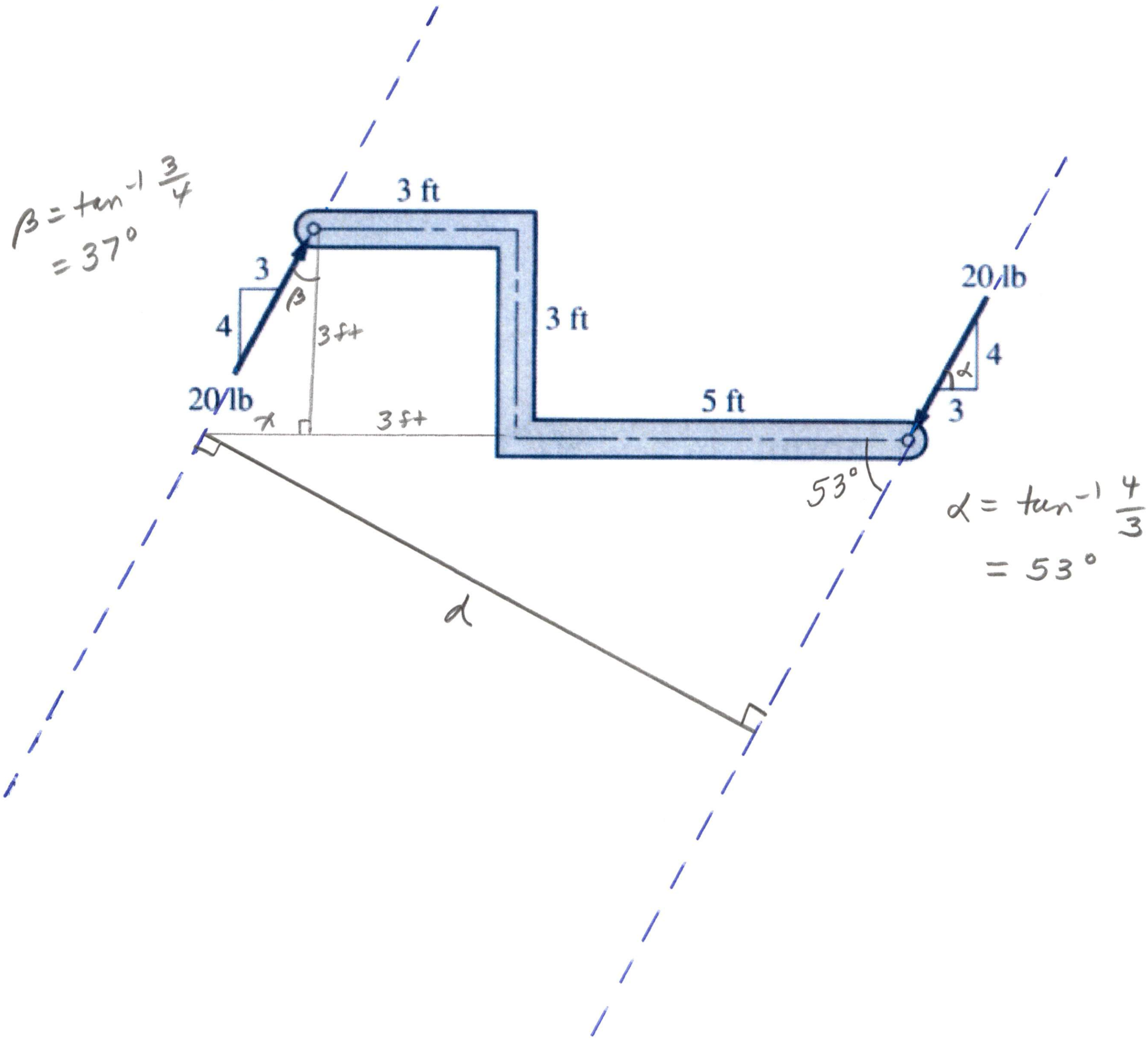


$$M_{\not{c}} = -12\,lb(3\,ft) + -16\,lb(8\,ft)$$

$$= -36\,lb \cdot ft - 128\,lb \cdot ft$$

$$= \underline{\underline{-164\,lb \cdot ft}}$$

Solution.
Use the definition directly



Find x

$$\tan 37° = \frac{x}{3\,ft}$$

$$x = 3\,ft \tan 37° = 2.3\,ft$$

Find d

$$\sin 53° = \frac{d}{10.3\,ft}$$

$$d = 10.3\,ft \sin 53° = 8.2\,ft$$

$$M_{\phi} = -20\,lb(8.2\,ft) = \underline{\underline{-164\,lb \cdot ft}}$$

2–51 Replace the 600-lb force acting on the connection in Fig. P2–51 with an equivalent force-couple system at the center of rivet B.

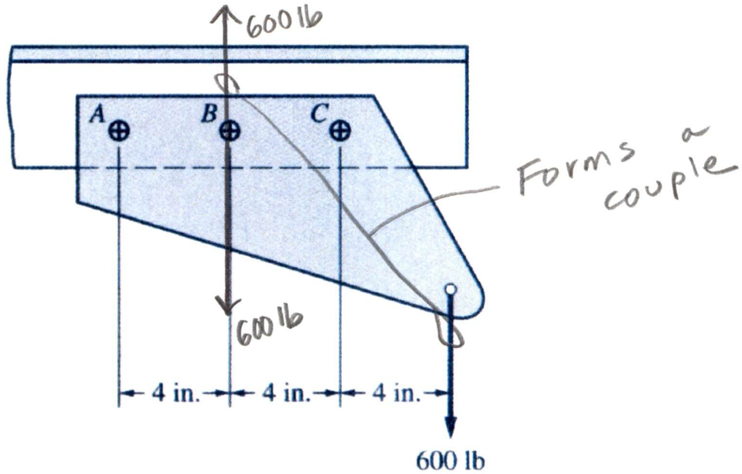


FIGURE P2–51

$$M_{\not{c}} = -600 \text{ lb } (8 \text{ in})$$
$$= -4800 \text{ lb} \cdot \text{in}$$

and $M_{\not{c}} = -400 \text{ lb} \cdot \text{ft}$

"Equivalent"

$\equiv$