

Chapter 3 - Equilibrium of Coplanar Force Systems

Reading: Chapter 3 - Pages 103 - 123

3-1

Introduction

Statics deals primarily with the equilibrium of structures or machines.

Loads

During the design process, it's essential to consider the anticipated structural load of a project. Loads are commonly understood as forces that cause stresses, deformations, or accelerations. These loads are applied to a structure or its components that cause stress or displacement. All structures must withstand loads or they will fail and/or fall apart. In order to build a structure to the correct tolerances, you need to know what kinds of external forces will affect it.

Permanent Structures

The design of structures, such as buildings and bridges, is typically performed by a structural engineer or an architect. The majority of structures designed and constructed for the built environment are permanent and intended to last for decades.

Temporary Structures

During the construction phase temporary structures, such as concrete formwork, falsework/shoring, and scaffolding, are typically erected to temporarily support loads or to provide access areas to parts of the building that are too high for workers to reach.

Concepts of Equilibrium and Free Body Diagrams

Whether a structure is designed and built to be permanent or used temporarily, the concepts of static equilibrium and the importance of free-body diagrams are fundamental to understanding how they are designed for reliable and safe performance.

3-2

Equilibrium Equations

Key Concept:

Equilibrium – the forces acting on a rigid body impart no translational or rotational motion

Conditions of Equilibrium:

$$\sum F_x = 0$$

$$\sum F_y = 0$$

$$\sum M_A = 0 \text{ (about any point)}$$

These equations can be used to determine unknown forces applied to the rigid body or unknown reactions exerted on it by its supports.

The three equations can be used to solve for no more than three unknowns.

The Free-Body Diagram

Free-Body Diagram

In solving rigid body equilibrium problems, we must:

- Consider all forces acting on the body.
- Exclude any force that is not directly applied to the body.

Constructing an appropriate free-body diagram is the single most important step for the solution of mechanics problems. Correct solution of a statics problem always depends on the successful completion of the free-body diagram.

Constructing a Free-Body Diagram

Choose the free body to be used. This body is then detached from the ground and separated from all other bodies. The contour of the isolated free body is sketched.

Weight

The weight of the body represents the gravitational attraction of the earth. The weight of a body always acts vertically downward through the *center of gravity*.

Applied Forces

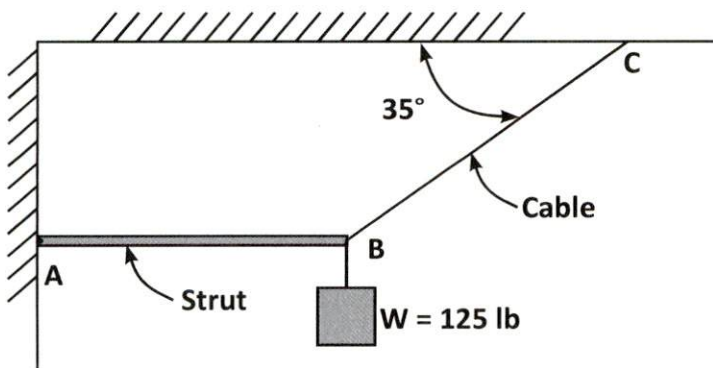
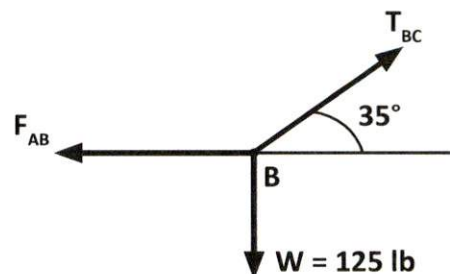
The applied forces include the loads that are applied to the free body. When these forces are shown on the free-body diagram, their magnitudes, directions, and locations must be indicated clearly.

Reactions

The reactions represent the constraining forces exerted in the free body by the supports or by the connected bodies. Reactions due to the supports depend on the type of support. The three most common types of supports are rollers, rockers, cables, and links; hinge and pin; and fixed supports.

Dimensions and Angles

A free-body diagram must include all significant dimensions and angles necessary for specifying the direction and location of the forces. Internal forces within the body must never be drawn on the free-body diagram.

Force SystemFBD

Types of Supports

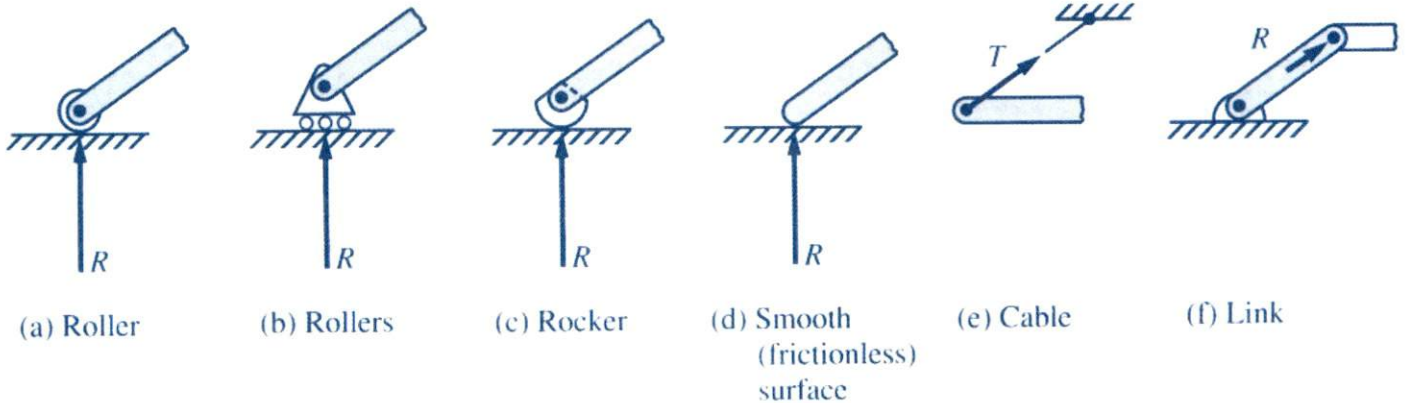
Roller Supports with One Unknown Reaction Element

Reactions equivalent to a force with a known line of action.

Examples:

Rollers, rockers, frictionless surfaces, short links and cables, collars on frictionless rods, frictionless pins in slots.

- Each can prevent motion in one direction only.
- Each involves one unknown (the magnitude of the reaction).
- The line of action is known and should be indicated on the free body diagram

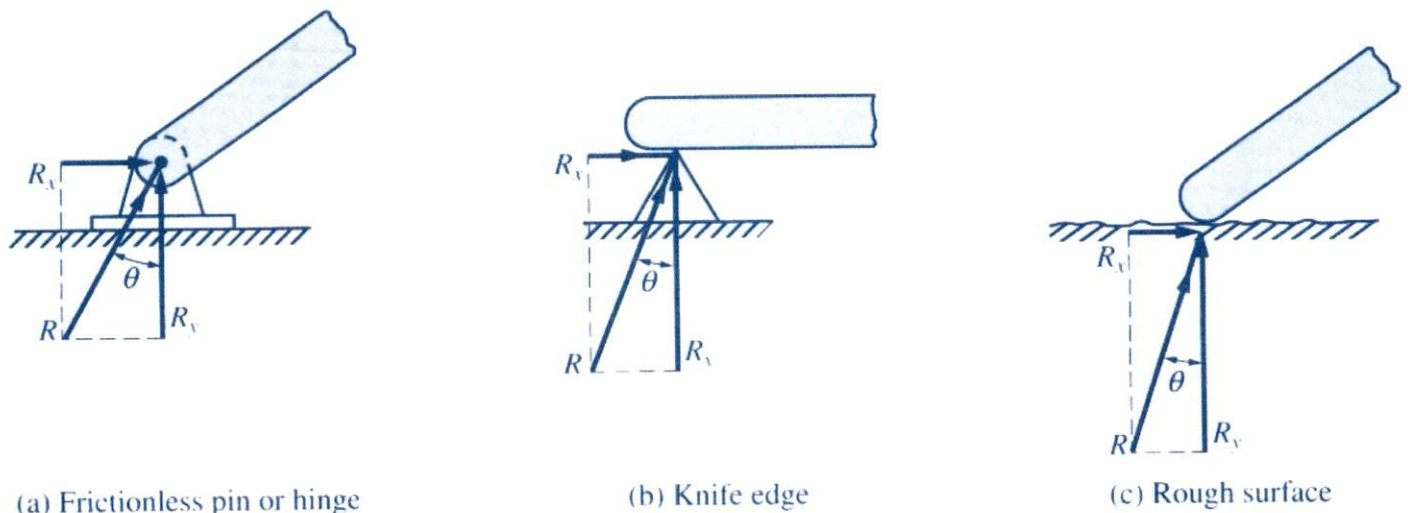
**Hinge Support with Two Unknown Reaction Elements**

Reactions equivalent to a force of unknown direction and magnitude.

Examples:

Frictionless pins fitted in holes, hinges, rough surfaces.

- Each can prevent translation of the free body in all directions.
- They can not prevent rotation about the connection.
- Reactions of this type have two unknowns and are represented by their x & y components.



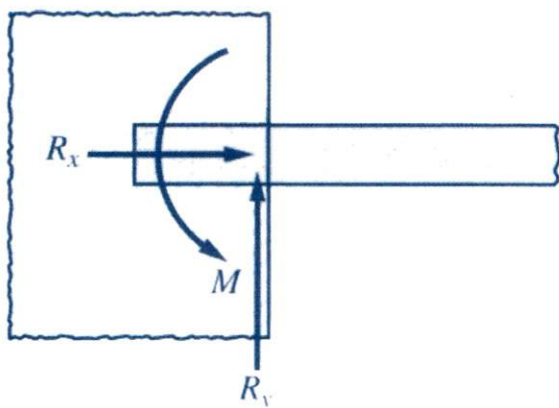
Fixed Support with Three Unknown Reaction Elements

Reactions equivalent to a force and a moment.

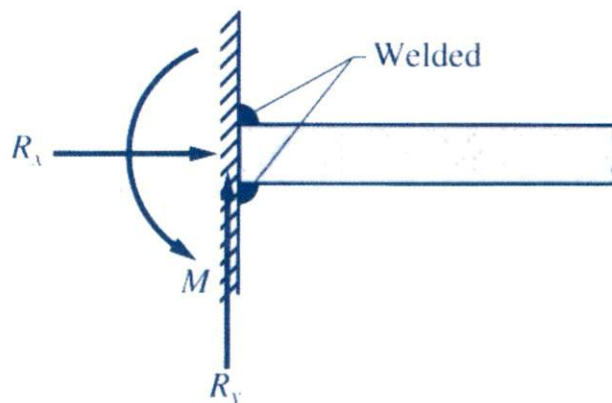
Examples:

Fixed support's which oppose any motion of the free body and thus constrain it completely.

- Fixed supports produce forces over the entire surface of contact that can be reduced to a force and a moment.
- Reactions of this type have three unknowns, represented by the 2 components of the force and the moment.



(a) Built-in supports



(b) Fixed support

Note:

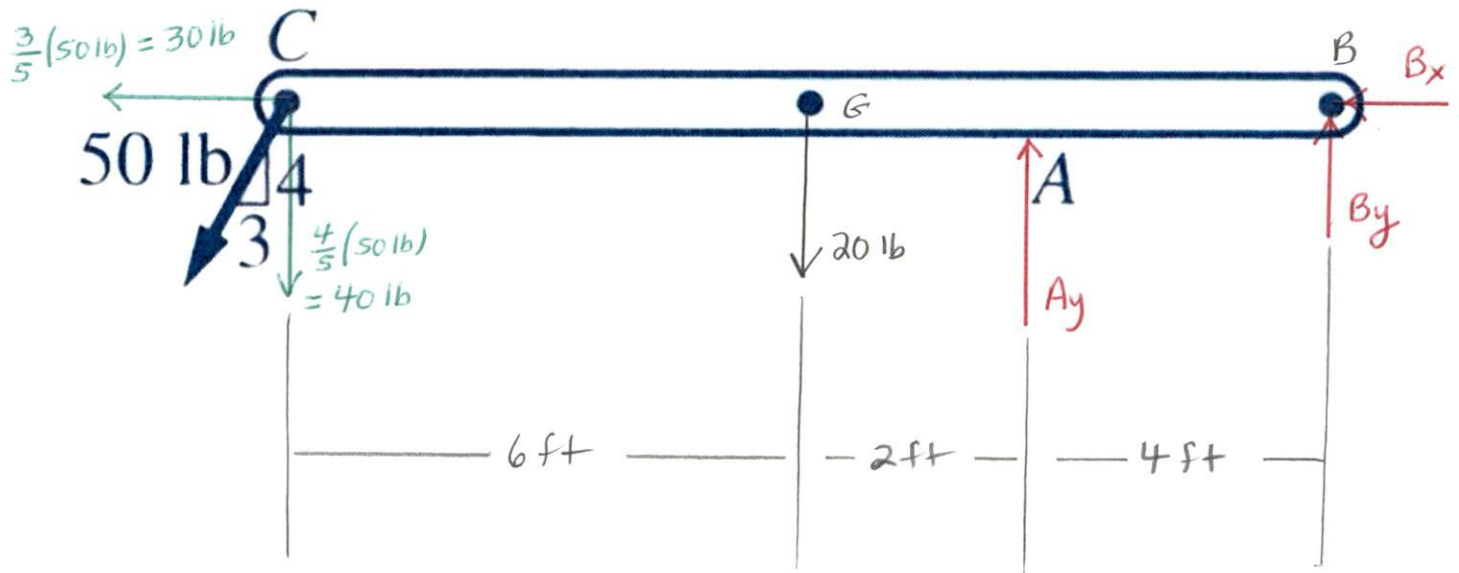
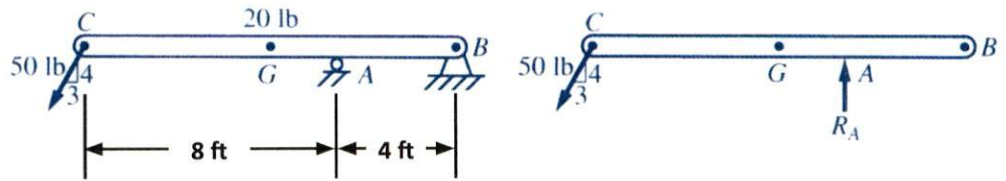
When the sense (direction) of an unknown force or moment is not readily apparent, no attempt should be made to determine it. The sense should be arbitrarily assigned and the sign of the answer used to confirm whether or not the original assumption was correct.

Sketching the Free Body Diagram (FBD)

Examples

Problem	Description	Body	Incomplete FBD
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3-1 Uniform beam of 20-lb weight supported by roller at A and hinge at B.



FBD

How many unknowns are there? 3

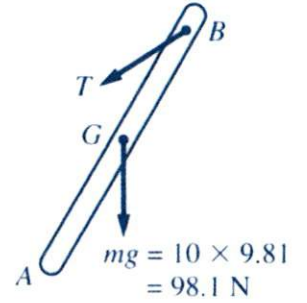
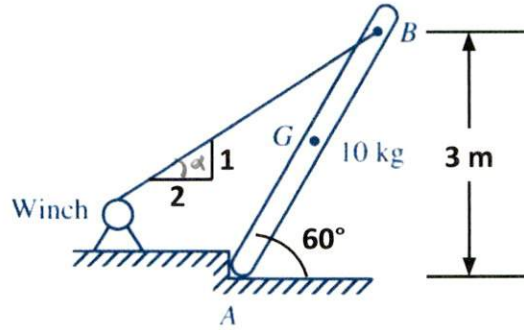
How many unknowns can we solve for? 3

$$\left. \begin{aligned} \sum F_x = 0 \\ \sum F_y = 0 \\ \sum M_A = 0 \end{aligned} \right\} \begin{array}{l} \text{Three} \\ \text{Equilibrium} \\ \text{Equations} \end{array}$$

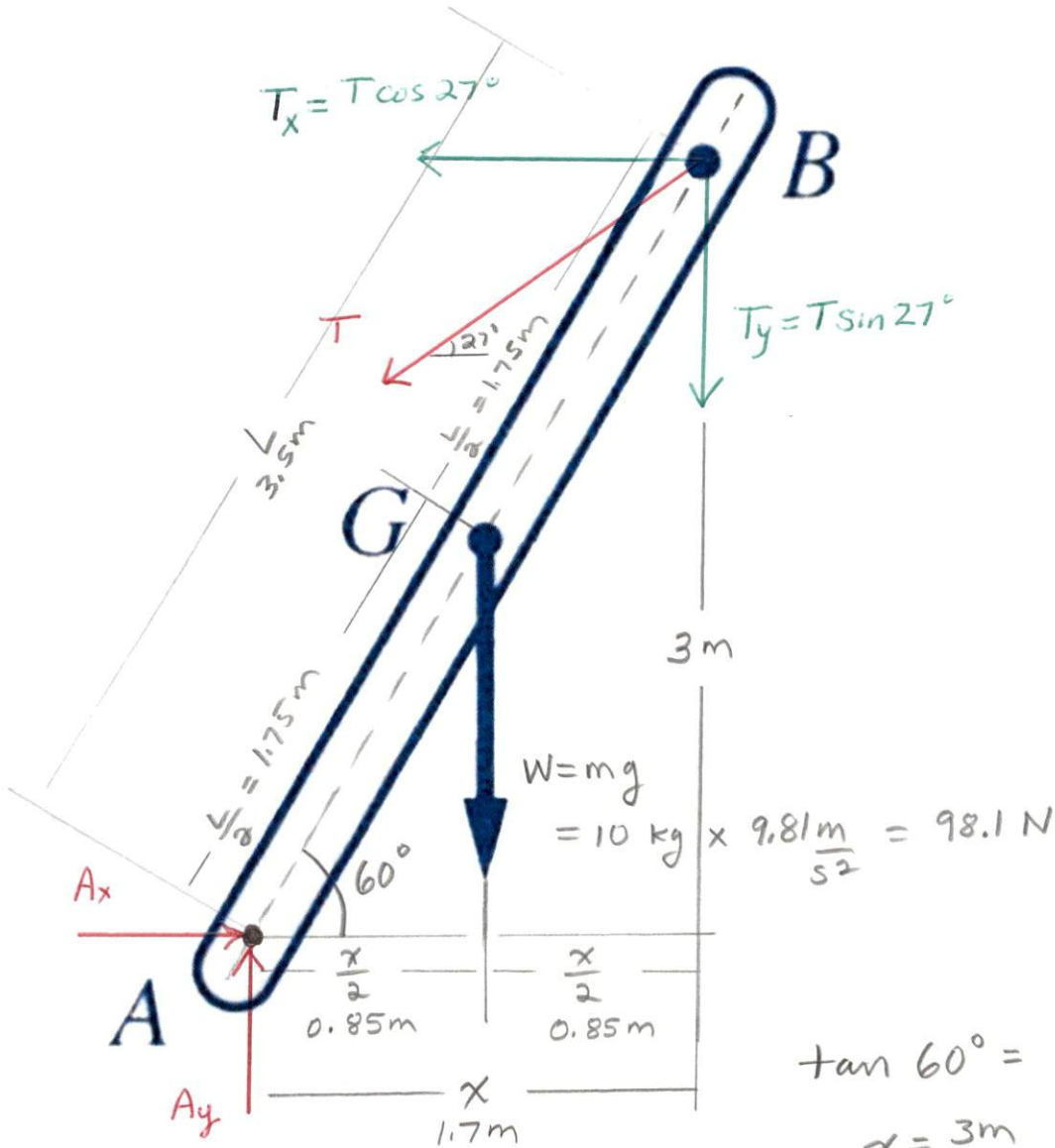
The directions for the 3 unknown reactions at the supports are "Guesses". Mathematics will tell us if we "Guessed" correctly or not.

3-2

Uniform pole of 10-kg mass being hoisted into position by a winch.



$$\alpha = \tan^{-1} \frac{1}{2} = 27^\circ$$



FBD

$$\sin 60^\circ = \frac{3 \text{ m}}{L}$$

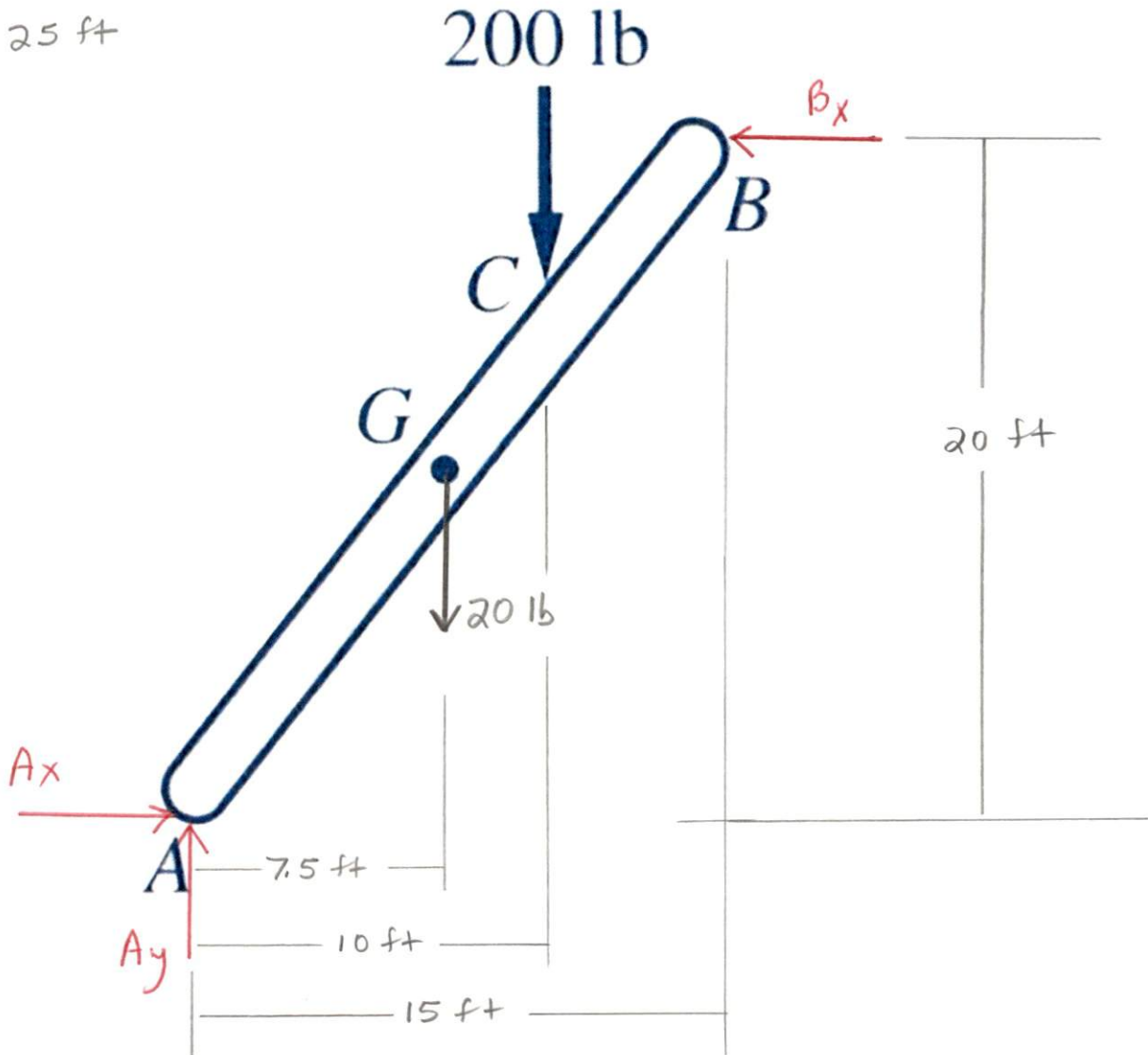
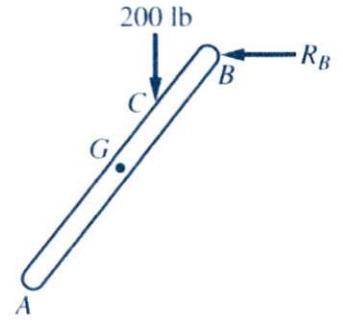
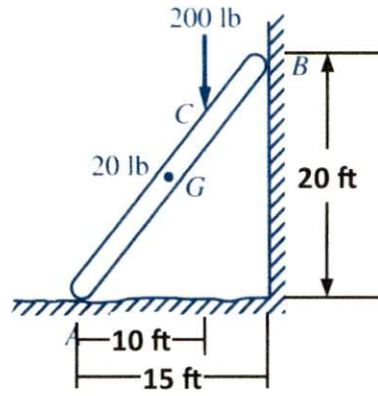
$$L = \frac{3 \text{ m}}{\sin 60^\circ} = 3.5 \text{ m}$$

3-3

A 20-lb ladder supported by a smooth wall and a rough floor.

What is the length of the ladder?

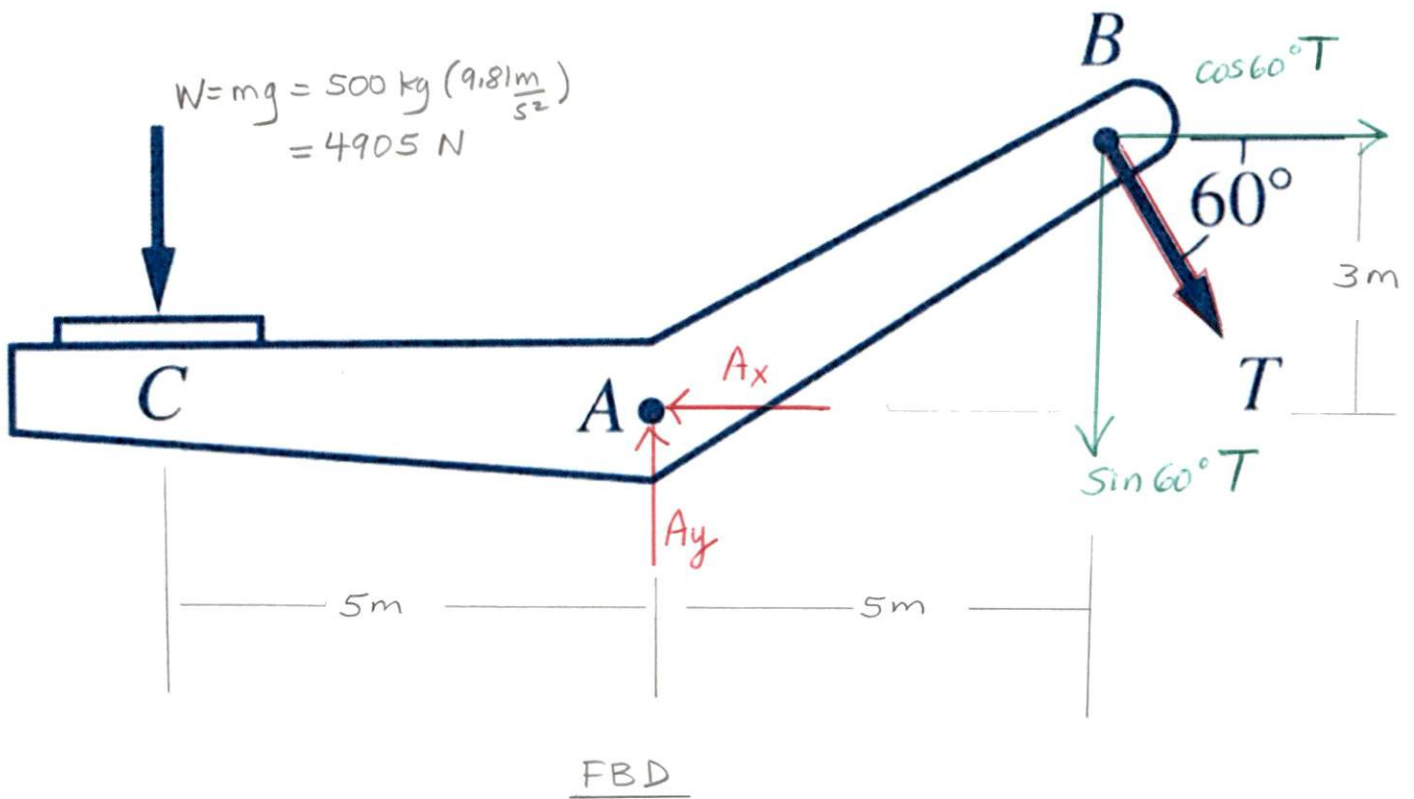
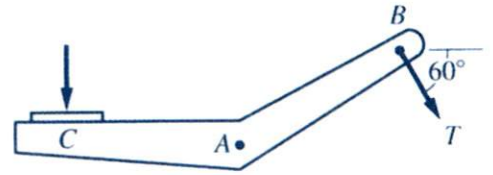
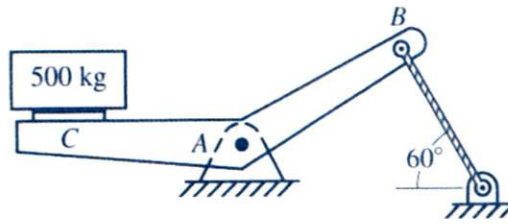
$$L = \sqrt{15 \text{ ft}^2 + 20 \text{ ft}^2} = 25 \text{ ft}$$



FBD

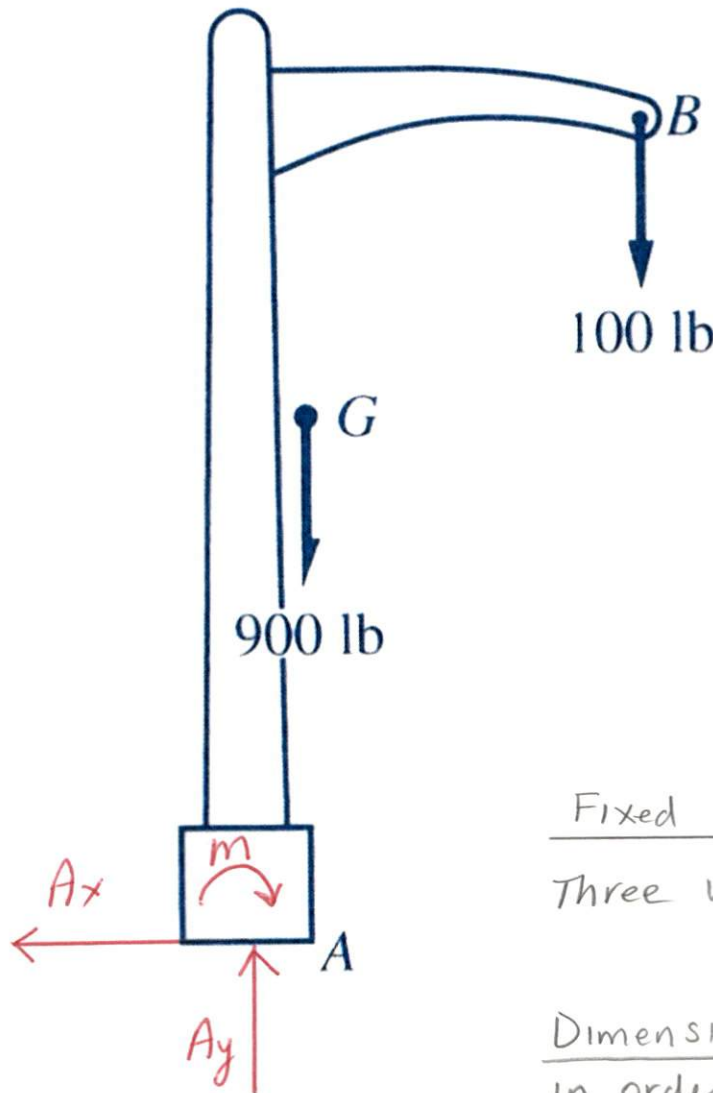
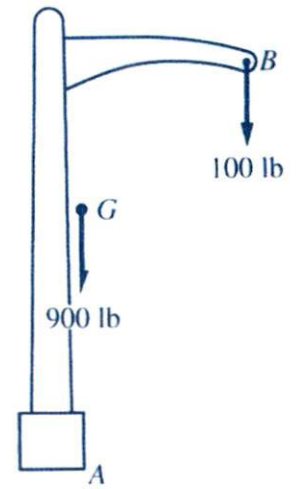
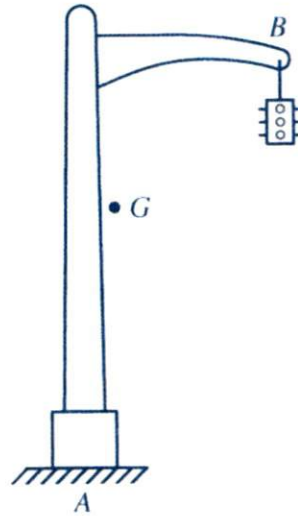
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Bell crank of negligible mass holding 500-kg mass supported by a pin at A and a cable at B.



3-5

Traffic-signal pole weighing 900 lb with a 100-lb traffic signal supported by a fixed support at A.

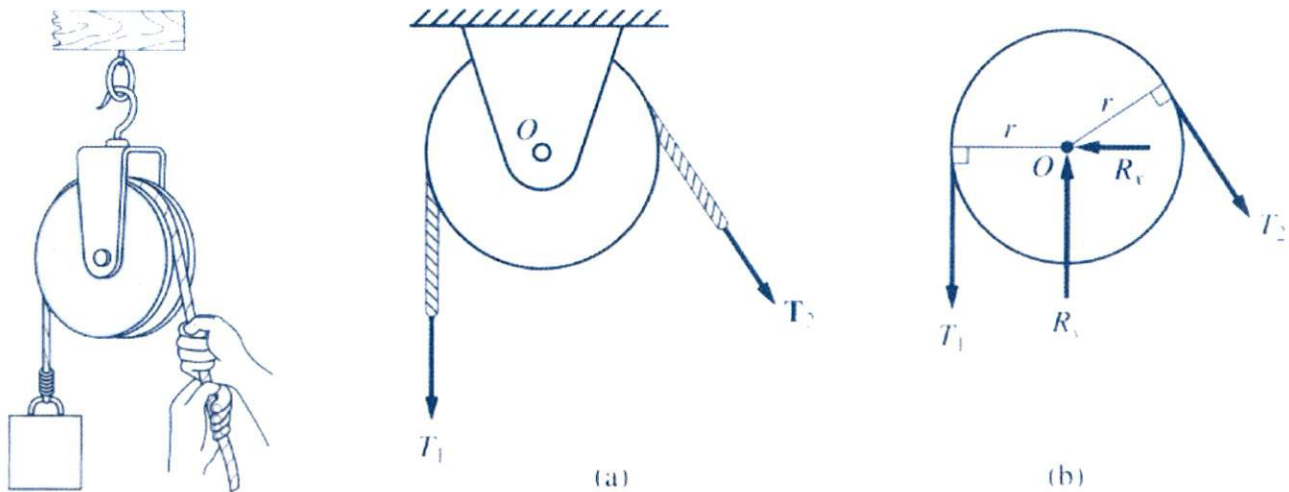


Fixed Support

Three unknown Reactions

Dimensions are needed
in order to solve for
the reactions at the
Fixed Support at A.

Support Reactions – Pulley



Equilibrium Equation $\sum M_O = 0$, is

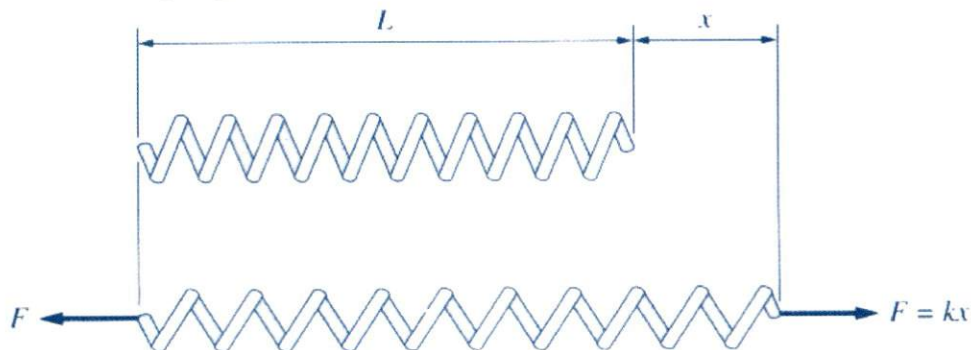
$$\sum M_O = T_1(r) - T_2(r) = 0$$

From which we get

$$T_1 = T_2$$

If the axle of a pulley is frictionless and the pulley is in equilibrium, the magnitudes of the tensile forces of the rope on the two sides of the pulley are the same.

Support Reactions - Linear Spring



Spring:

- Elastic element capable of developing elastic restoring forces when deformed.
- Linear spring develops a tensile or compressive force proportional to the amount of elongation or contraction of the spring.
- Free or undeformed length is the length of the spring in its undeformed state.
- Deformation (elongation or contraction) of a spring is the change in the length of the spring from its free length.

Equation of Tensile Force:

$$F = kx$$

Where, K is the spring constant or stiffness (units: N/m and lb/ft or lb/in)
 x is the elongation from a free length L in the linear spring (units: m and ft)