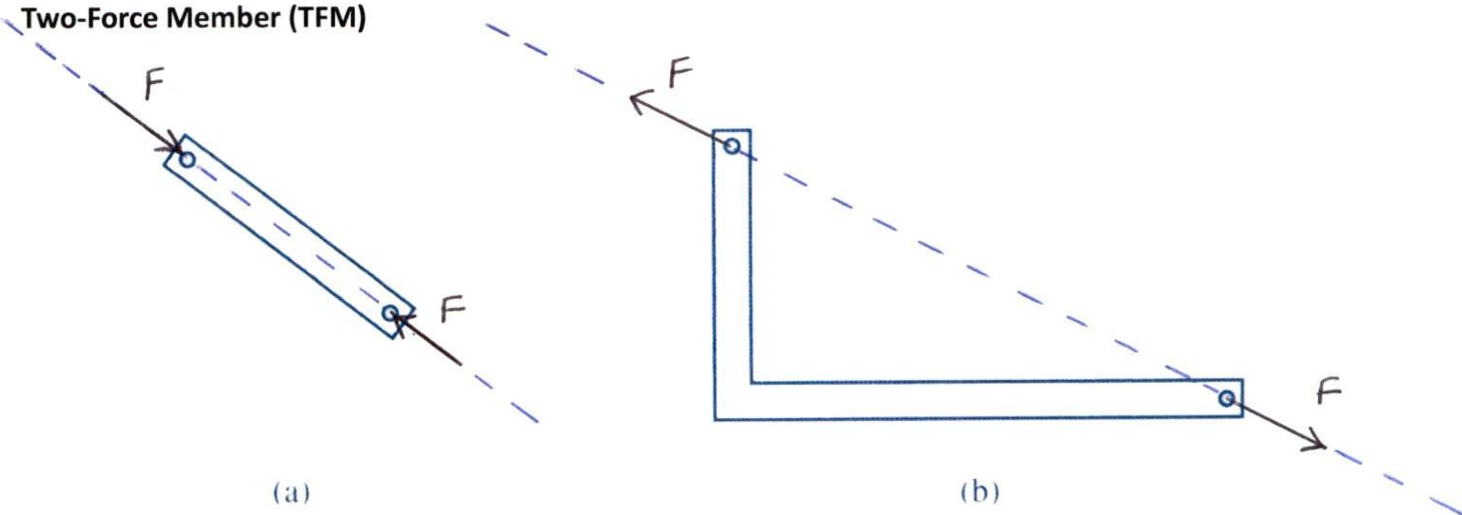
3-5
Equilibrium of Concurrent Coplanar Force Systems

Key Concept:
Equilibrium – the forces impart no translational or rotational motion

Conditions of Equilibrium:
$\sum F_x = 0$
$\sum F_y = 0$

These two equilibrium equations can be used to determine two unknown forces applied to the rigid body or unknown reactions exerted on it by its supports.

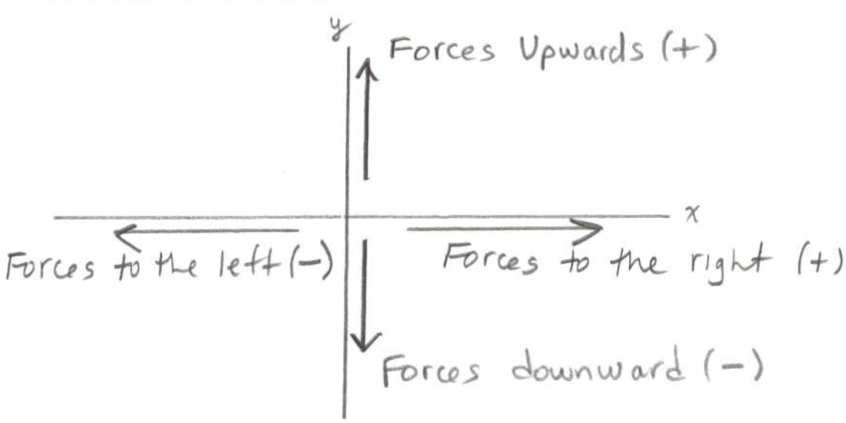
Two-Force Member (TFM)



(a) (b)

Equilibrium conditions require that the two forces be equal in magnitude, opposite in direction, and acting along the line joining the two points of application.

Sign Convention (Review)



Equilibrium Equations

$\Sigma F_x = 0$

$\Sigma F_y = 0$

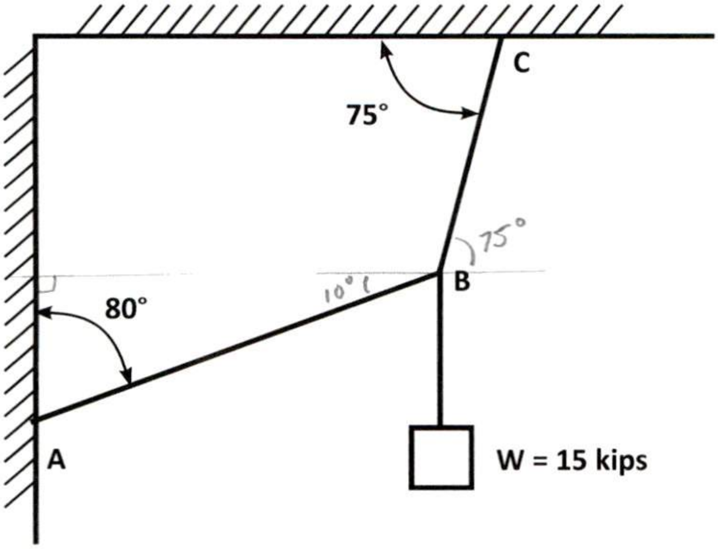
$\Sigma M_A = 0$

Moments

CCW +M

CW −M

Example #1. Determine the tension in the cables AB and BC (a) mathematically from the force triangle and (b) by the method of rectangular components and equilibrium equations

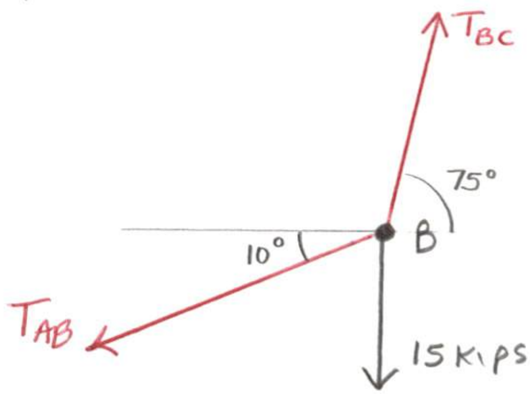


If three concurrent forces are in equilibrium, they must form a closed <u>Force Triangle</u>.

Solution.

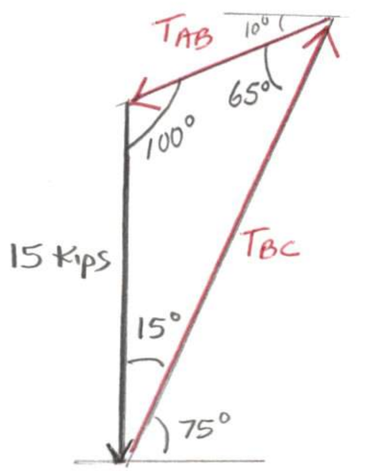
(a) mathematically from the force triangle (Head-to-Tail, all forces)

Step 1. Draw the FBD!



<u>FBD</u>

Step 2. Draw the Force-Triangle and solve using Trig.



<u>Force-Triangle</u>

<u>Law of Sines</u>

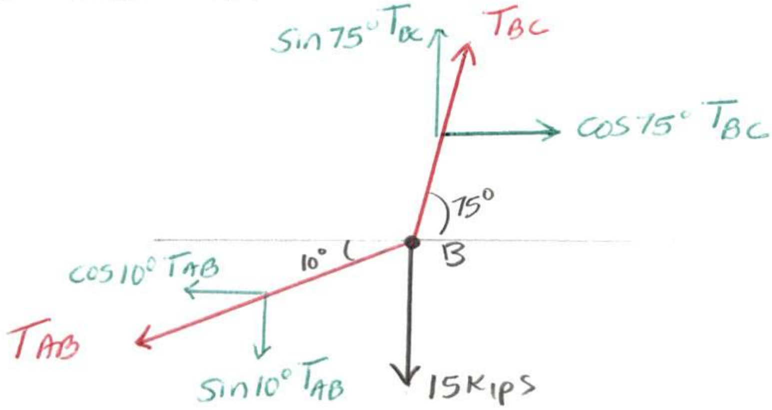
$$\frac{T_{AB}}{\sin 15^\circ} = \frac{T_{BC}}{\sin 100^\circ} = \frac{15 \text{ kips}}{\sin 65^\circ}$$

$$T_{AB} = \frac{\sin 15^\circ (15 \text{ kips})}{\sin 65^\circ} = \underline{\underline{4.3 \text{ kips}}}$$

$$T_{BC} = \frac{\sin 100^\circ (15 \text{ kips})}{\sin 65^\circ} = \underline{\underline{16.3 \text{ kips}}}$$

(b) by the method of rectangular components and equilibrium equations

Step 1. Draw the FBD!



FBD

Step 2. Equilibrium Equations

$[\Sigma F_x = 0]$ $\quad -\cos 10°\, T_{AB} + \cos 75°\, T_{BC} = 0 \quad (1)$

$[\Sigma F_y = 0]$ $\quad -\sin 10°\, T_{AB} + \sin 75°\, T_{BC} - 15 \text{ kips} = 0 \quad (2)$

From (1) $\quad \cos 75°\, T_{BC} = \cos 10°\, T_{AB}$

$$T_{BC} = \frac{\cos 10°\, T_{AB}}{\cos 75°} \quad (3)$$

Subst (3) into (2)

$$-\sin 10°\, T_{AB} + \sin 75° \left[\frac{\cos 10°\, T_{AB}}{\cos 75°}\right] = 15 \text{ kips}$$

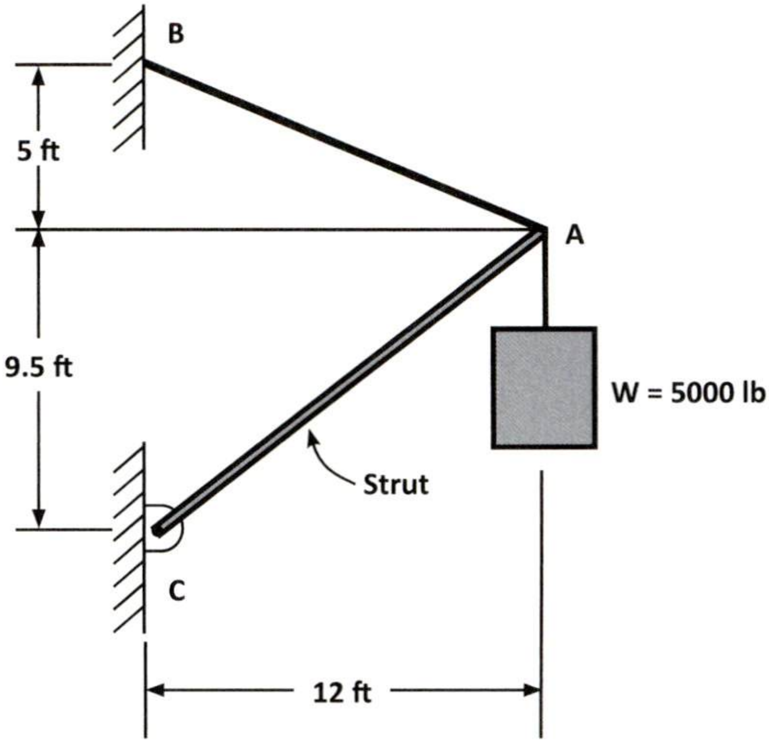
$$T_{AB} = \frac{15 \text{ kips}}{-\sin 10° + \dfrac{\sin 75° \cos 10°}{\cos 75°}} = \underline{\underline{4.3 \text{ kips}}}$$

From (3)

$$T_{BC} = \frac{\cos 10° (4.3 \text{ kips})}{\cos 75°} = \underline{\underline{16.3 \text{ kips}}}$$

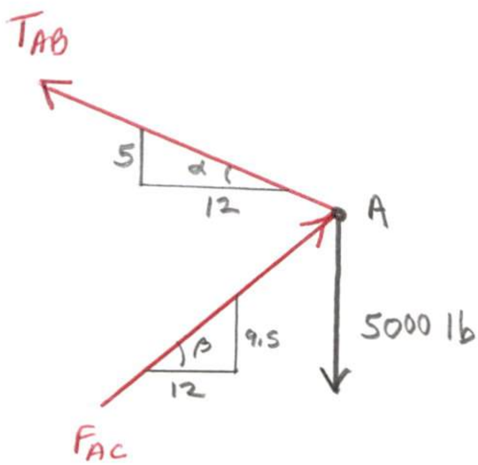
Example #2

Determine the tension in the cable AB and the axial force in the strut AC (a) mathematically from the force triangle and (b) by equilibrium equations.



Solution.

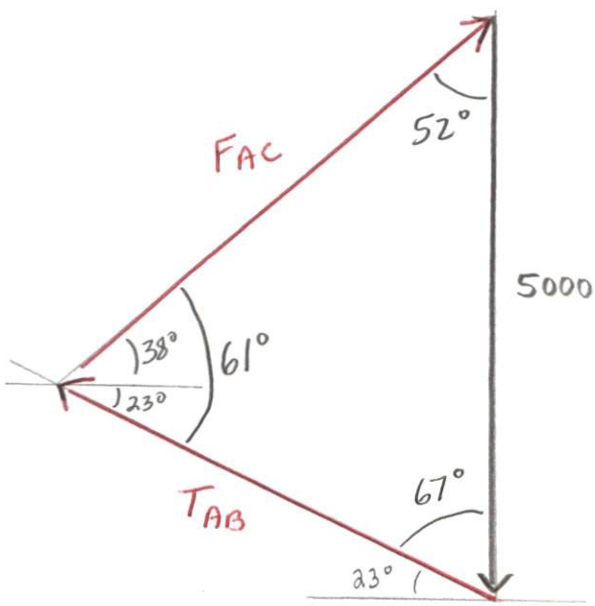
(a) mathematically from the force triangle



FBD

$$\alpha = \tan^{-1}\frac{5}{12} = 23^\circ$$

$$\beta = \tan^{-1}\frac{9.5}{12} = 38^\circ$$



Force Triangle

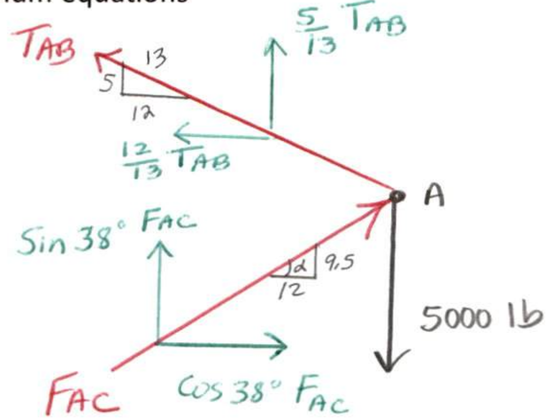
Law of Sines

$$\frac{F_{AC}}{\sin 67^\circ} = \frac{T_{AB}}{\sin 52^\circ} = \frac{5000\ \text{lb}}{\sin 61^\circ}$$

$$F_{AC} = \frac{\sin 67^\circ (5000\ \text{lb})}{\sin 61^\circ} = 5262\ \text{lb}$$

$$T_{AB} = \frac{\sin 52^\circ (5000\ \text{lb})}{\sin 61^\circ} = 4505\ \text{lb}$$

(b) by equilibrium equations



$$\alpha = \tan^{-1} \frac{9.5}{12} = 38°$$

FBD

Equilibrium Equations

$$[\Sigma F_x = 0] \qquad -\frac{12}{13} T_{AB} + \cos 38° F_{AC} = 0 \qquad (1)$$

$$[\Sigma F_y = 0] \qquad \frac{5}{13} T_{AB} + \sin 38° F_{AC} - 5000 \text{ lb} = 0 \qquad (2)$$

Cramer's Rule

$$D = \begin{vmatrix} -\frac{12}{13} & \cos 38° \\ \frac{5}{13} & \sin 38° \end{vmatrix} = -0.57 - 0.3 = -0.87$$

$$D_x = \begin{vmatrix} 0 & \cos 38° \\ 5000 & \sin 38° \end{vmatrix} = -3940$$

$$D_y = \begin{vmatrix} -\frac{12}{13} & 0 \\ \frac{5}{13} & 5000 \end{vmatrix} = -4615$$

$$T_{AB} = \frac{D_x}{D} = \frac{-3940}{-0.87} = \underline{\underline{4529}} \text{ lb}$$

$$F_{AC} = \frac{D_y}{D} = \frac{-4615}{-0.87} = \underline{\underline{5304}} \text{ lb}$$

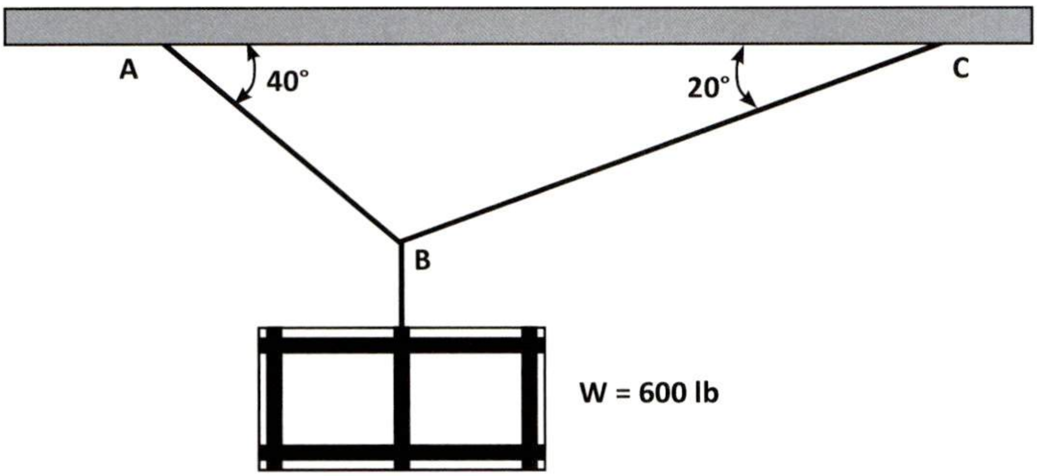
Example #3

A crate weighing 600 lb is suspended by two cables.

(a) Draw a free body diagram of point

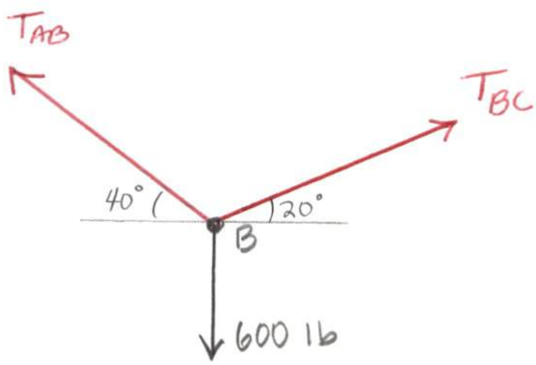
(b) Construct a force triangle that satisfies the conditions of equilibrium of point B

(c) Use the force triangle to determine the tension in cables AB and BC



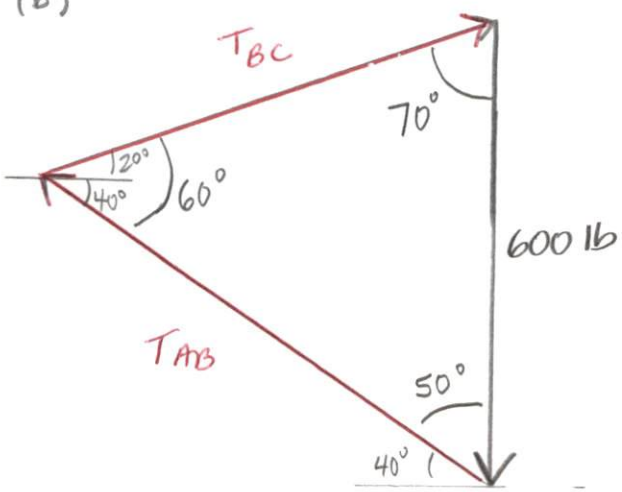
Solution.

(a) FBD



FBD

(b)



Force Triangle

(c) Law of Sines

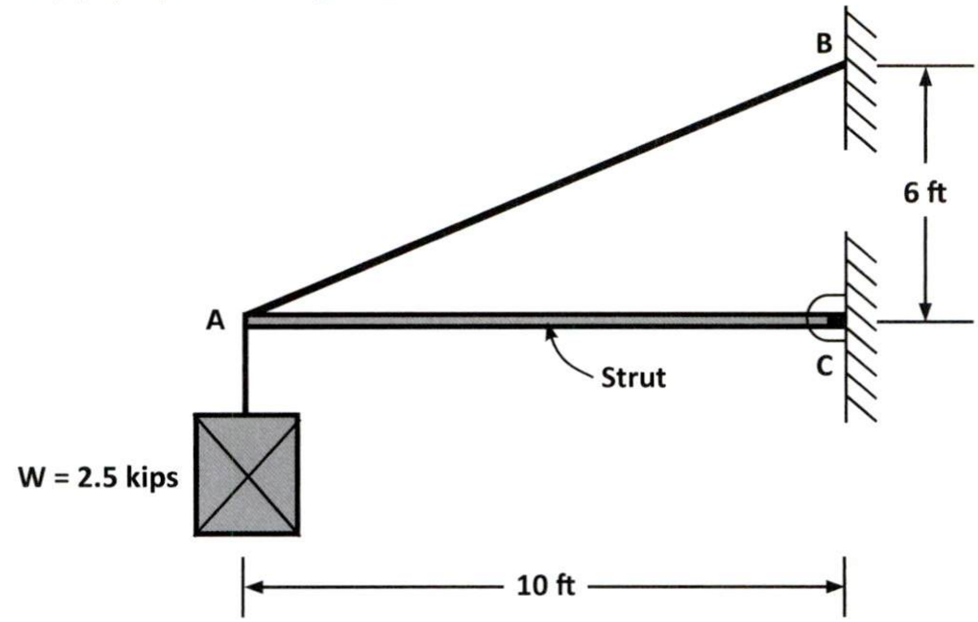
$$\frac{T_{BC}}{\sin 50^\circ} = \frac{T_{AB}}{\sin 70^\circ} = \frac{600\text{ lb}}{\sin 60^\circ}$$

$$T_{BC} = \frac{\sin 50^\circ (600\text{ lb})}{\sin 60^\circ} = \underline{531\text{ lb}}$$

$$T_{AB} = \frac{\sin 70^\circ (600\text{ lb})}{\sin 60^\circ} = \underline{651\text{ lb}}$$

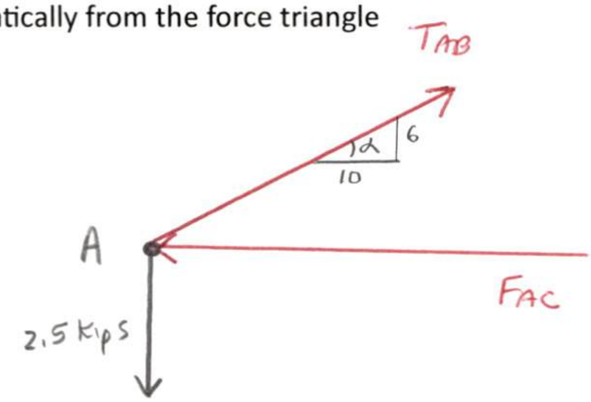
Example #4

Determine the tension in the cable and the tension or compression in the strut (a) mathematically from the force triangle and (b) by equilibrium equations.



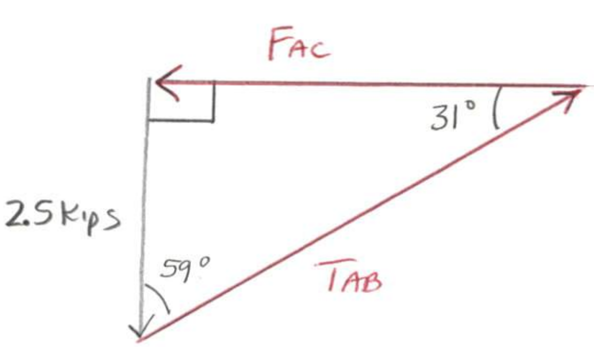
Solution.

(a) mathematically from the force triangle



FBD

$$\alpha = \tan^{-1}\frac{6}{10} = 31°$$



Force Triangle

Right-Triangle Math!

$$\tan 31° = \frac{2.5\text{ kips}}{F_{AC}}$$

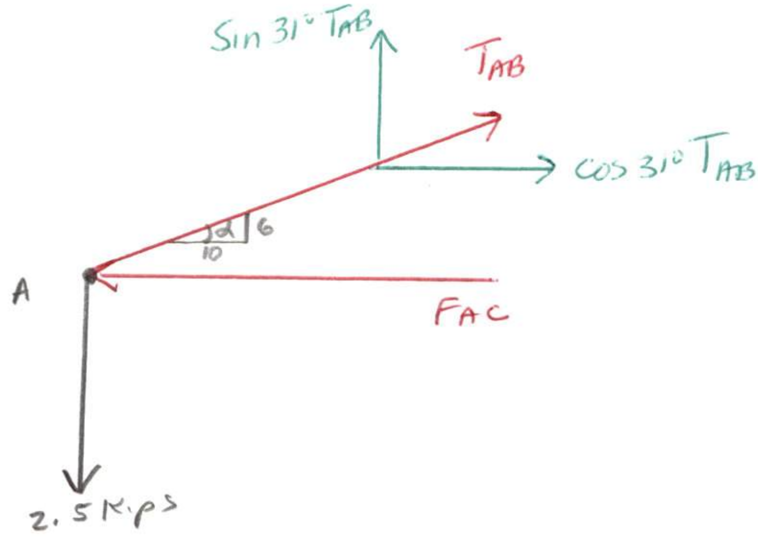
$$F_{AC} = \frac{2.5\text{ kips}}{\tan 31°} = \underline{\underline{4.2\text{ kips}}}$$

From the Pythagorean Theorem

$$T_{AB} = \sqrt{2.5\text{ kips}^2 + 4.2\text{ kips}^2} = \underline{\underline{4.9\text{ kips}}}$$

(b) by equilibrium equations

$$\alpha = \tan^{-1}\frac{6}{10} = 31^\circ$$



FBD

Equilibrium Equations

$[\Sigma F_x = 0]$ $\quad - F_{AC} + \cos 31^\circ\, T_{AB} = 0 \qquad (1)$

$[\Sigma F_y = 0]$ $\quad \sin 31^\circ\, T_{AB} - 2.5\text{ kips} = 0 \qquad (2)$

From (2) $\quad \sin 31^\circ\, T_{AB} = 2.5\text{ kips}$

$$T_{AB} = \frac{2.5\text{ kips}}{\sin 31^\circ} = \underline{\underline{4.9}}\text{ kips}$$

From (1) $\quad F_{AC} = \cos 31^\circ\,(4.9\text{ kips}) = \underline{\underline{4.2}}\text{ kips}$

Example #5

Determine the push force P required to hold the 10-kg homogeneous cylinder stationary on the 10° incline shown in Fig. P3-11. Neglect the weight of the handle.

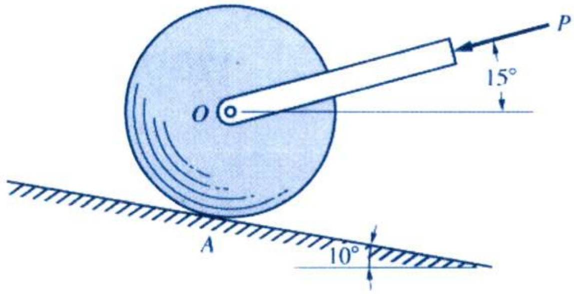
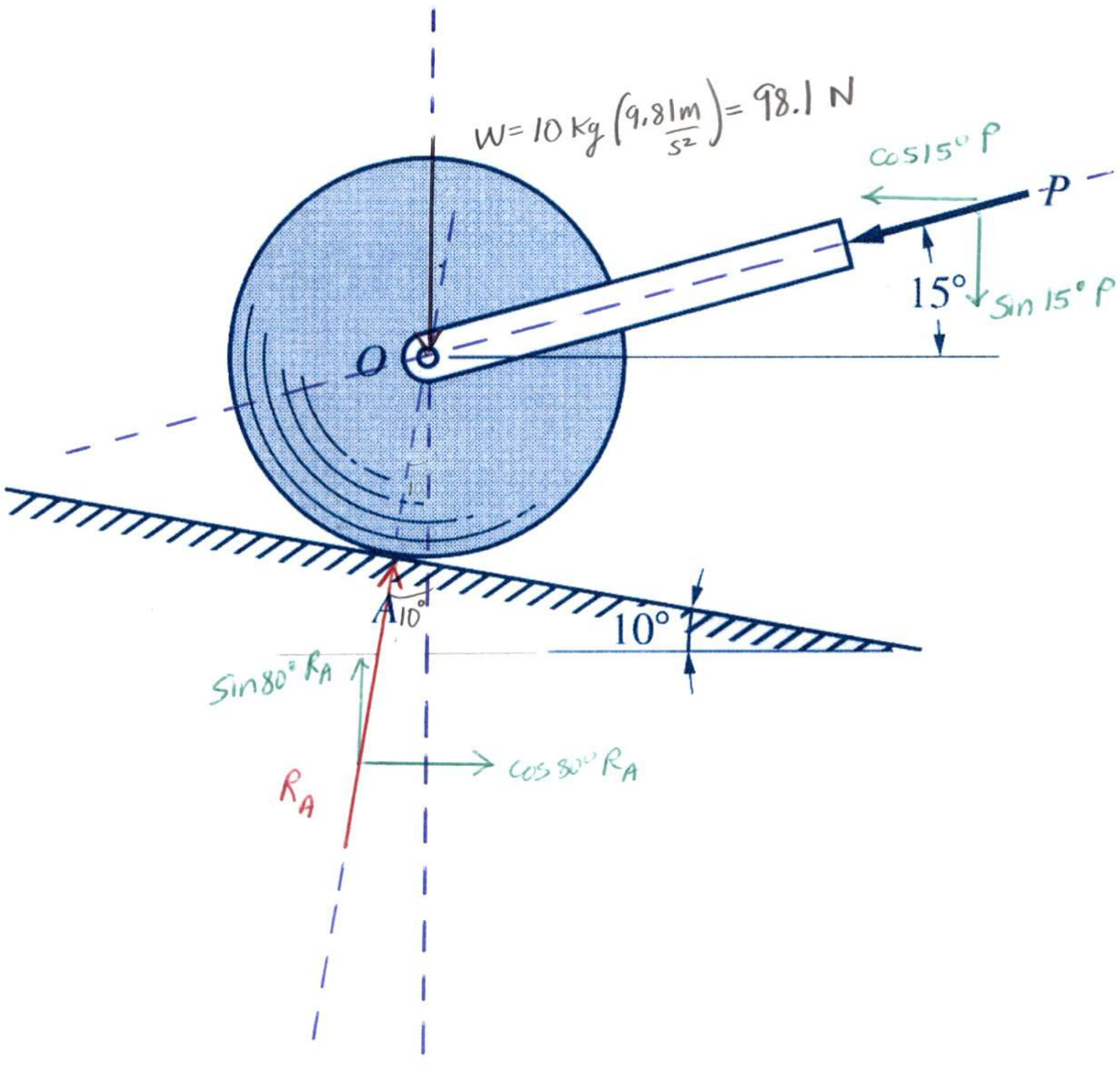


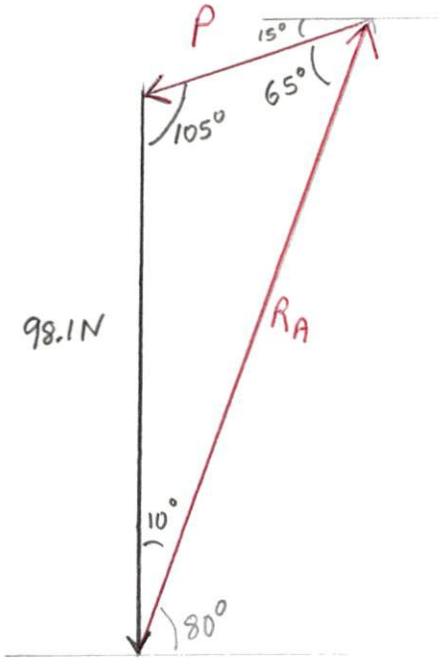
FIGURE P3–11

Solution.



FBD

Force Triangle



$$\frac{P}{\sin 10^\circ} = \frac{98.1N}{\sin 65^\circ}$$

$$P = \frac{\sin 10^\circ (98.1N)}{\sin 65^\circ} = \underline{\underline{18.8\ N}}$$

Equilibrium Equations

$[\Sigma F_x = 0]$ $\quad -\cos 15^\circ P + \cos 80^\circ R_A = 0 \quad (1)$

$[\Sigma F_y = 0]$ $\quad -\sin 15^\circ P + \sin 80^\circ R_A - 98.1N = 0 \quad (2)$

From (1) $\quad R_A = \dfrac{\cos 15^\circ P}{\cos 80^\circ} \quad (3)$

Subst. (3) into (2)

$$-\sin 15^\circ P + \sin 80^\circ \left(\frac{\cos 15^\circ P}{\cos 80^\circ} \right) = 98.1\ N$$

$$P = \frac{98.1N}{\left[\sin 15^\circ + \dfrac{\sin 80^\circ \cos 15^\circ}{\cos 80^\circ} \right]} = \underline{\underline{18.8\ N}}$$