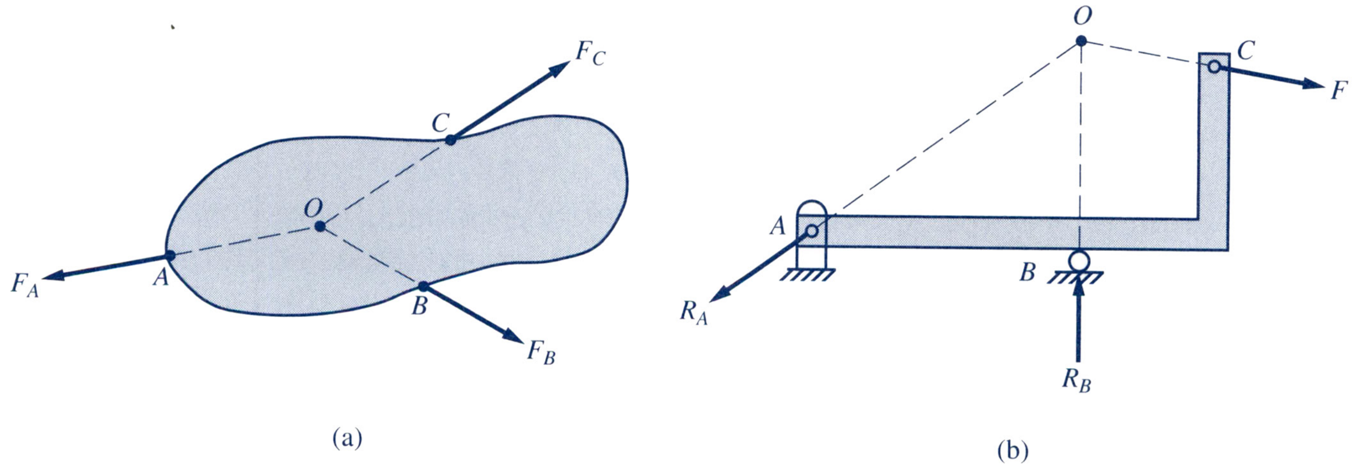
Three-Force Body



Equilibrium conditions require that the three forces be coplanar and concurrent.
Exception: The three forces are parallel.

3–39 See Fig. P3–39. Determine the reactions of the supports at A and D due to the 400-lb load applied to the frame shown by (a) the force triangle and (b) equilibrium equations.

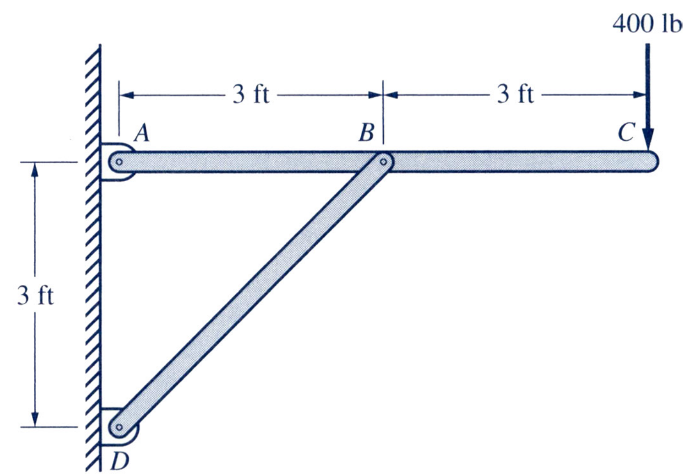
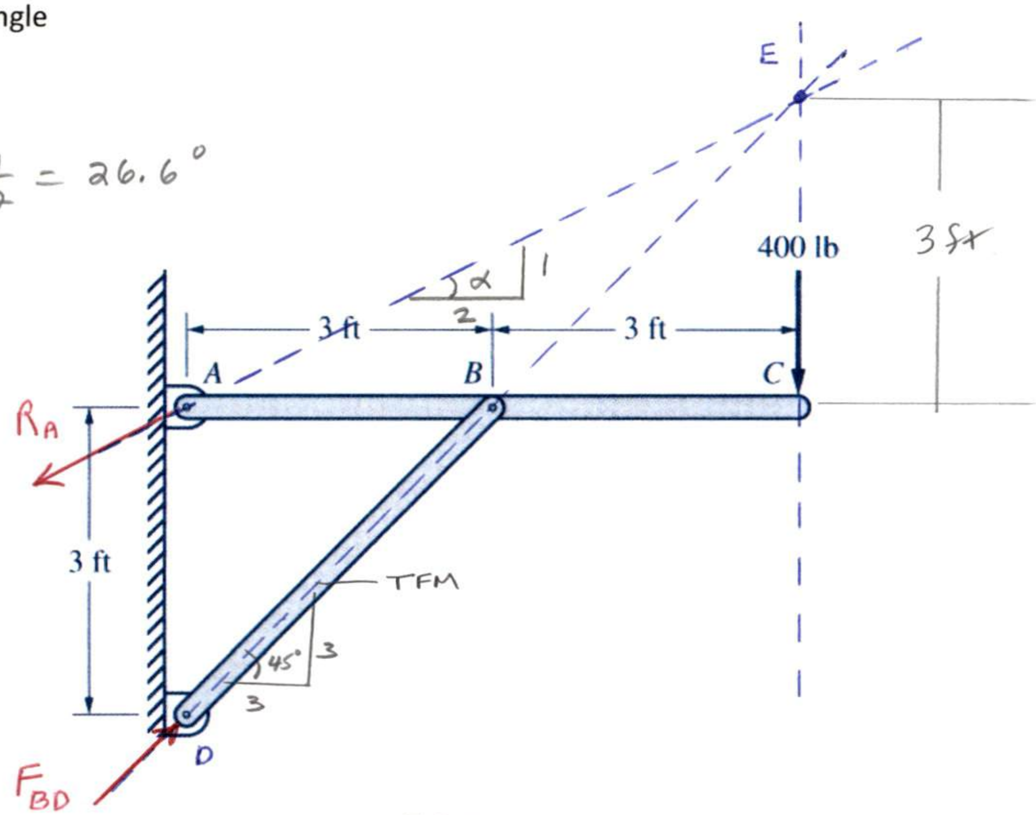


FIGURE P3–39

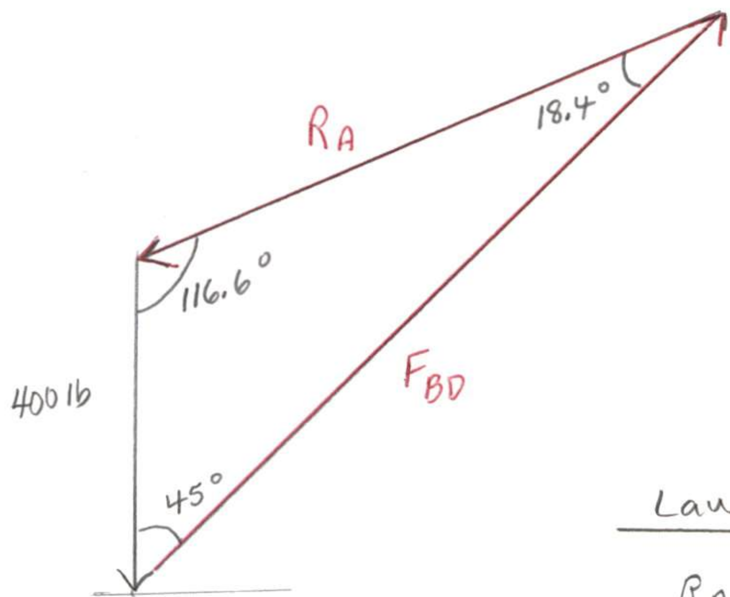
Solution. See next page

(a) the force triangle

$$\alpha = \tan^{-1}\frac{1}{2} = 26.6^\circ$$



FBD



Force Triangle

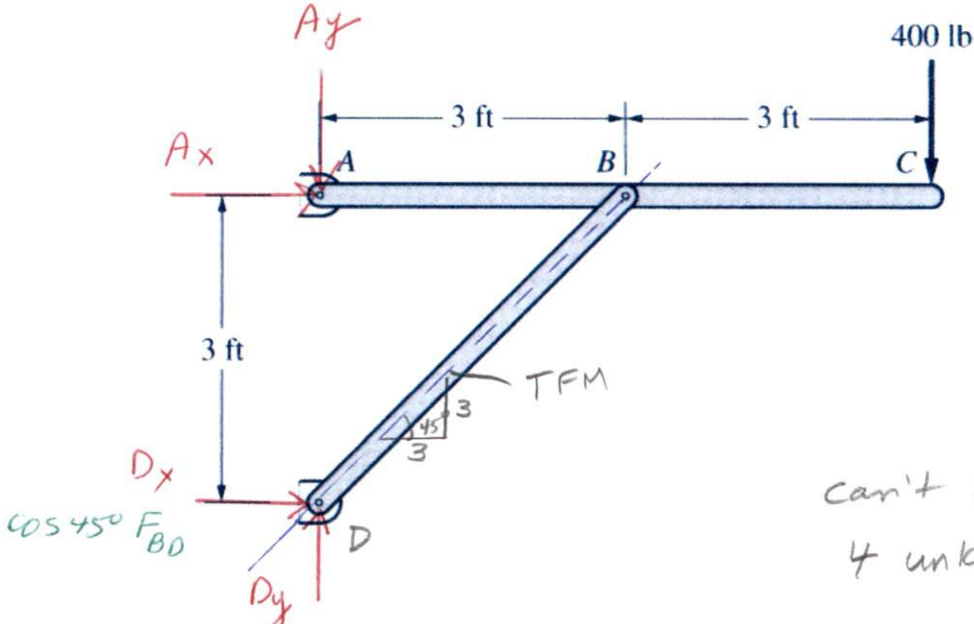
Law of Sines

$$\frac{R_A}{\sin 45^\circ} = \frac{F_{BD}}{\sin 116.6^\circ} = \frac{400 \text{ lb}}{\sin 18.4^\circ}$$

$$R_A = \frac{\sin 45^\circ (400 \text{ lb})}{\sin 18.4^\circ} = 896 \text{ lb} \angle 206.6^\circ$$

$$F_{BD} = \frac{\sin 116.6^\circ (400 \text{ lb})}{\sin 18.4^\circ} = 1133 \text{ lb} \angle 45^\circ$$

(a) equilibrium equations



FBD

ccw +M

cw −M

Can't Solve for 4 unknowns!

Must use the fact that member BD is a Two-Force Member

Equilibrium Equations

$[\Sigma M_A = 0]$ $\quad -400\text{ lb}\,(6\text{ ft}) + \cos 45°\, F_{BD}\,(3\text{ ft}) = 0$

$$F_{BD} = \frac{2400\text{ lb}\cdot\text{ft}}{\cos 45°\,(3\text{ ft})} = 1131\text{ lb} \angle 45°$$

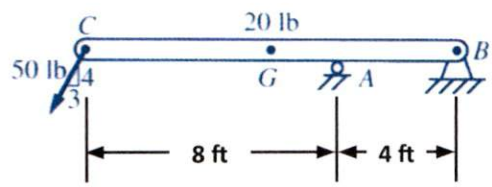
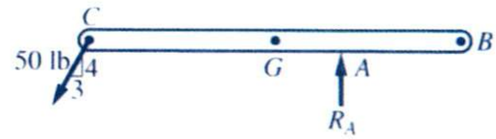
$[\Sigma F_x = 0]$ $\quad A_x + \cos 45°\, F_{BD} = 0$

$A_x = -800\text{ lb} \longrightarrow$ and $\boxed{A_x = 800\text{ lb} \leftarrow}$

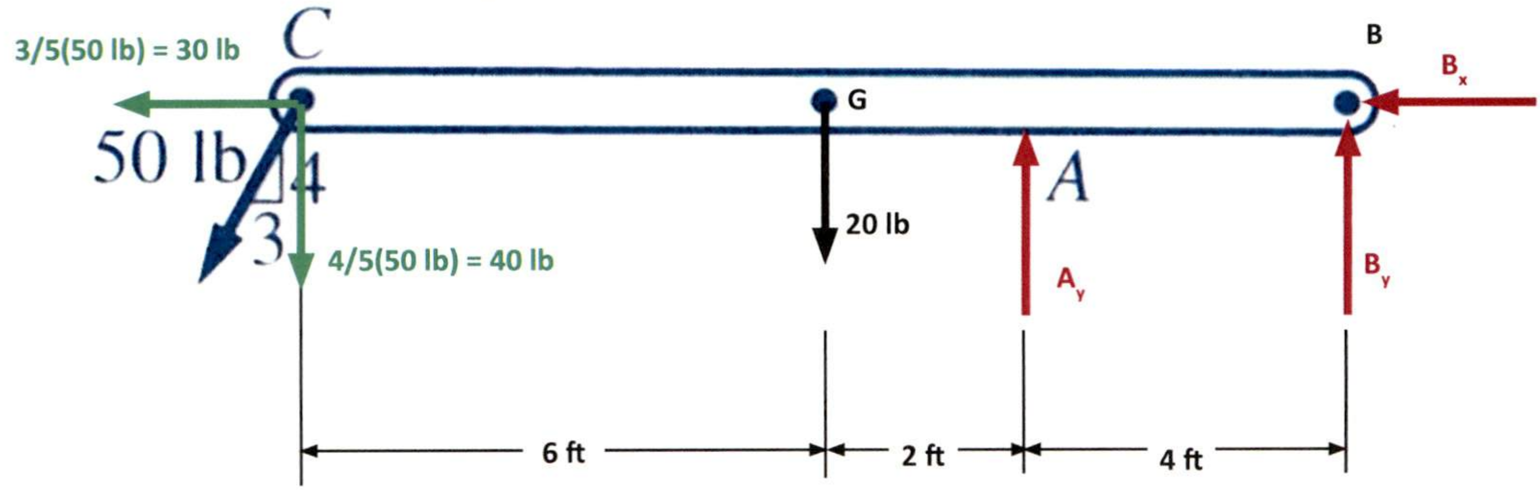
$[\Sigma F_y = 0]$ $\quad -A_y - 400\text{ lb} + \sin 45°\, F_{BD} = 0$

$A_y = \sin 45°\,(1131\text{ lb}) - 400\text{ lb} = \underline{\underline{400\text{ lb} \downarrow}}$

$R_A = \sqrt{A_x^2 + A_y^2} = \sqrt{800\text{ lb}^2 + 400\text{ lb}^2} = 894\text{ lb}$

Problem	*Description*	*Body*	*Incomplete FBD*
3–1	Uniform beam of 20-lb weight supported by roller at *A* and hinge at *B*.		

Step 1. Draw the Free Body Diagram



FBD

ccw +M ↺
cw −M ↻

Step 2. Equilibrium Equations

$[\Sigma F_x = 0]$ $\quad -30\text{ lb} - B_x = 0$

$B_x = -30\text{ lb} \leftarrow$ and $\boxed{B_x = 30\text{ lb} \rightarrow}$

"Guessed" incorrectly

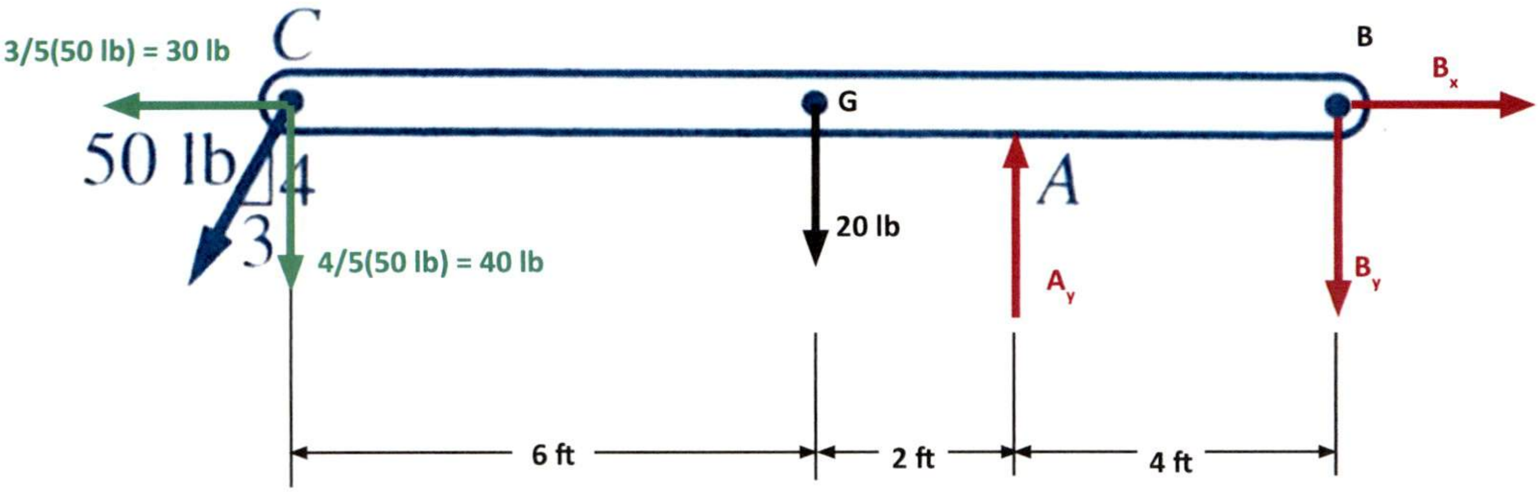
$[\Sigma M_A = 0]$ $\quad 40\text{ lb}(8\text{ ft}) + 20\text{ lb}(2\text{ ft}) + B_y(4\text{ ft}) = 0$

$$B_y = -\frac{360\text{ lb}\cdot\text{ft}}{4\text{ ft}} = -90\text{ lb} \uparrow$$

and $\boxed{B_y = 90\text{ lb} \downarrow}$

$[\Sigma F_y = 0]$ $\quad -40\text{ lb} - 20\text{ lb} + A_y + B_y = 0$

$$A_y = 60\text{ lb} - (-90\text{ lb}) = \underline{\underline{150\text{ lb}}} \uparrow$$



FBD

ccw +M ↺
cw −M ↻

Equilibrium Equations

$[\Sigma F_x = 0] \quad -30\text{ lb} + B_x = 0$

$$B_x = \underline{\underline{30\text{ lb}}} \rightarrow$$

$[\Sigma M_A = 0] \quad 40\text{ lb}(8\text{ ft}) + 20\text{ lb}(2\text{ ft}) - B_y(4\text{ ft}) = 0$

$$B_y = \frac{360\text{ lb}\cdot\text{ft}}{4\text{ ft}} = \underline{\underline{90\text{ lb}}} \downarrow$$

$[\Sigma F_y = 0] \quad -40\text{ lb} - 20\text{ lb} + A_y - B_y = 0$

$$A_y = 60\text{ lb} + 90\text{ lb} = \underline{\underline{150\text{ lb}}} \uparrow$$

Identify the type of support and indicate the equivalent reaction provided by the support.

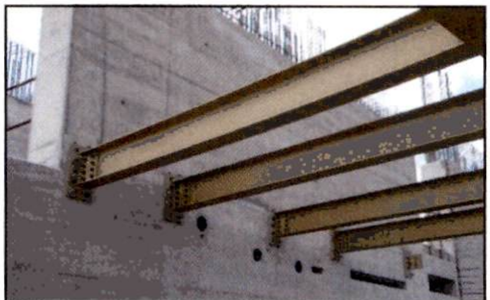
Roller $\uparrow$

Pin $\rightarrow \uparrow$

Simple $\uparrow$

Rocker $\uparrow$

Pin $\rightarrow \uparrow$



Fixed $\rightarrow \uparrow \circlearrowleft$

Fixed $\rightarrow \uparrow \circlearrowleft$

Roller $\uparrow$