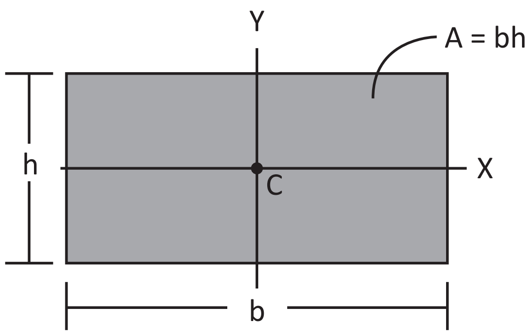
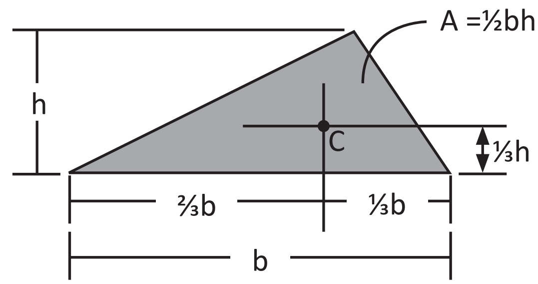
Chapter 7 - Center of Gravity and Centroids
Reading: Chapter 7 - Pages 259 - 284

7-1
Introduction



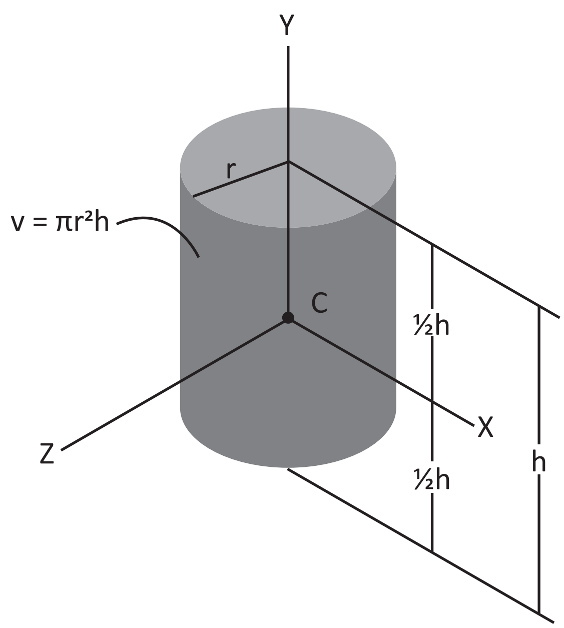
Rectangular area

The centroid, C, is a point defining the geometric center of an object.



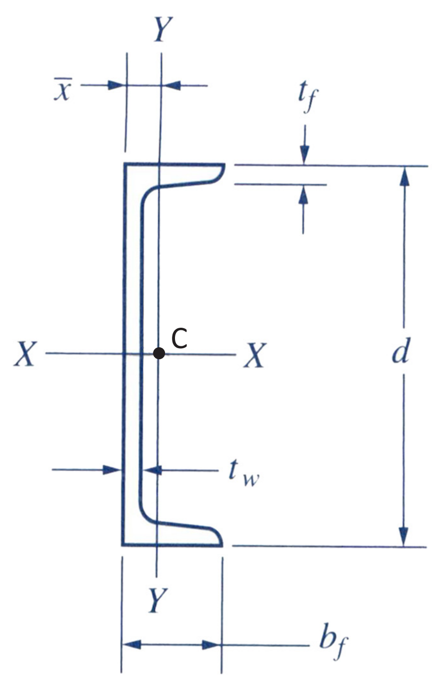
Triangular area

The center of gravity, G, is defined as a point about which the entire weight of the body is assumed to be concentrated. The centroid coincides with the center of mass or the center of gravity only if the material of the body is homogenous (density or specific weight is constant throughout the body).



Cylinder

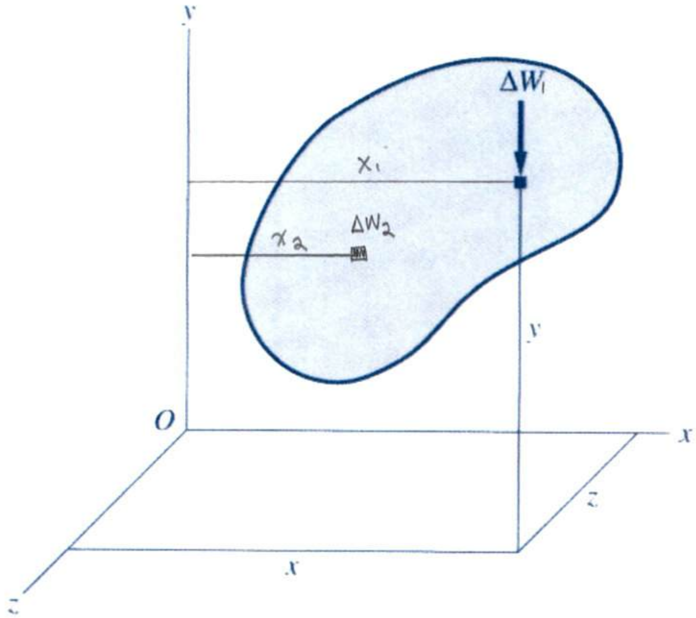
If an object has an axis of symmetry, then the centroid of the object lies on that axis.



Channel

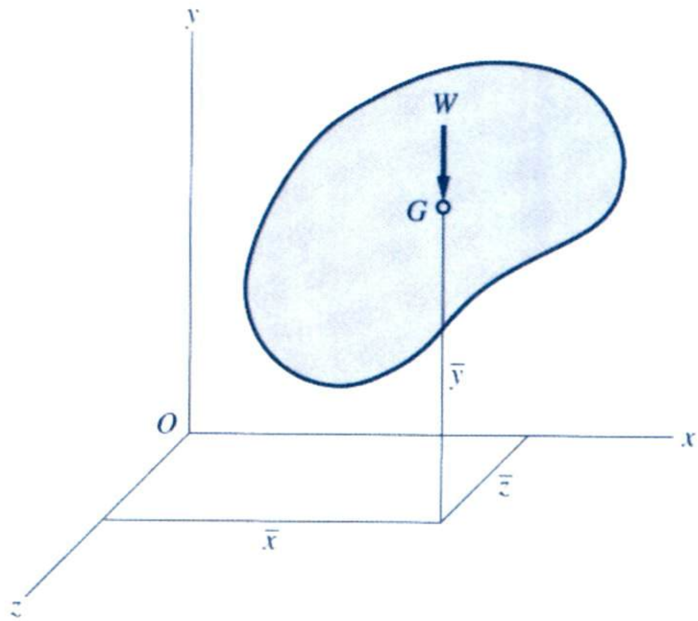
In some cases, the centroid may not be located on the object. For the C shape (American Standard Channel) structural steel component shown above the centroid is given by the X-X and Y-Y centroidal axes.

Center of Gravity (G)



(a)

W = Σ ΔW



(b)

G(x̄, ȳ, z̄)

The moment of W about O must be equal to the sum of the moments of the weight of all the particles about O.

Equating the moment about the z axis, we write

$\bar{x}\,W = \Sigma\, x\, \Delta W$

Equating the moment about the x axis, we write

$\bar{z}\,W = \Sigma\, z\, \Delta W$

Similarly, we can also write

$\bar{y}\,W = \Sigma\, y\, \Delta W$

The following equations can be used for locating the center of gravity, G, of a solid body:

$$\bar{x} = \frac{\Sigma x \Delta W}{\Sigma \Delta W} \qquad \bar{y} = \frac{\Sigma y \Delta W}{\Sigma \Delta W} \qquad \bar{z} = \frac{\Sigma z \Delta W}{\Sigma \Delta W} \qquad (7\text{–}1)$$

Centroid (C)
The center of mass of a geometric object of uniform density.

Locating the Centroid of a Volume
Homogeneous body (uniform)

γ = weight per unit volume (constant)

ΔW = γ Δv (weight of a particle)

W = γ v (weight of the entire body)

Substituting in Equation 7-1 and dividing through by γ, we obtain

$$\bar{x} = \frac{\Sigma x \Delta V}{\Sigma \Delta V} \qquad \bar{y} = \frac{\Sigma y \Delta V}{\Sigma \Delta V} \qquad \bar{z} = \frac{\Sigma z \Delta V}{\Sigma \Delta V} \qquad (7\text{–}2)$$

Centroids of Volumes of Common Shapes
Formulas for computing the volumes and locations of centroids of common geometric shapes, such as cylinders, spheres, hemispheres, circular cones, and so on, have been determined mathematically and are listed in Table 7-1 (see next page).

Centroid of Composite Volumes
A volume can be divided into several component volumes of shapes is called a composite volume. The coordinates of the centroid C ($\bar{x}$, $\bar{y}$, $\bar{z}$) of a composite volume may be computed from the following equations:

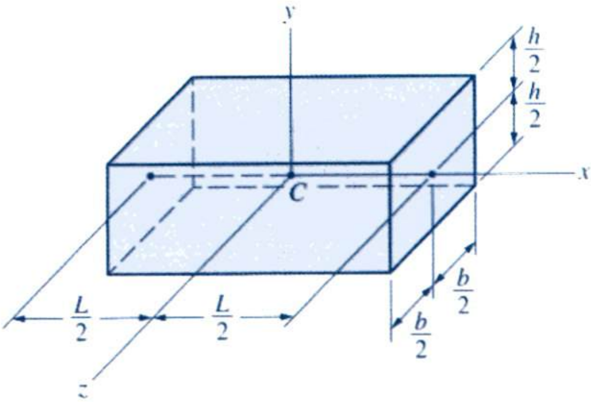
$$\bar{x} = \frac{\Sigma Vx}{\Sigma V} = \frac{V_1 x_1 + V_2 x_2 + V_3 x_3 + \cdots}{V_1 + V_2 + V_3 + \cdots}$$

$$\bar{y} = \frac{\Sigma Vy}{\Sigma V} = \frac{V_1 y_1 + V_2 y_2 + V_3 y_3 + \cdots}{V_1 + V_2 + V_3 + \cdots}$$

$$\bar{z} = \frac{\Sigma Vz}{\Sigma V} = \frac{V_1 z_1 + V_2 z_2 + V_3 z_3 + \cdots}{V_1 + V_2 + V_3 + \cdots} \qquad (7\text{–}3)$$

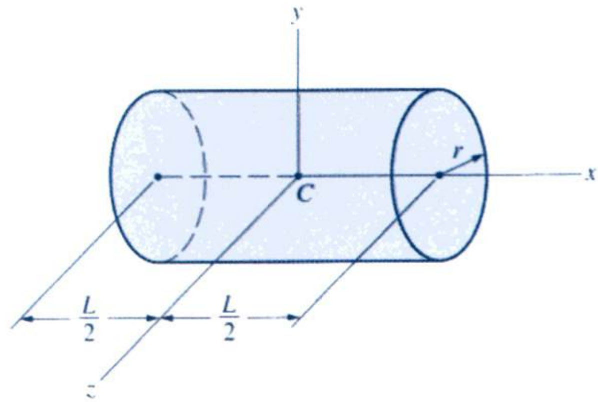
TABLE 7–1 Centroids of Volumes of Common Shapes

Rectangular Prism



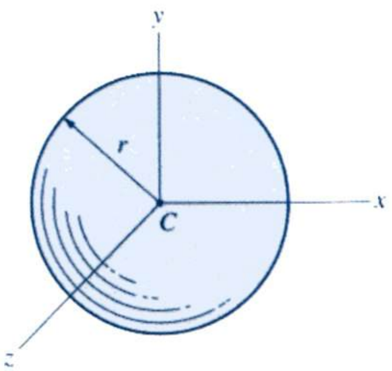
$V = bhL$

Cylinder



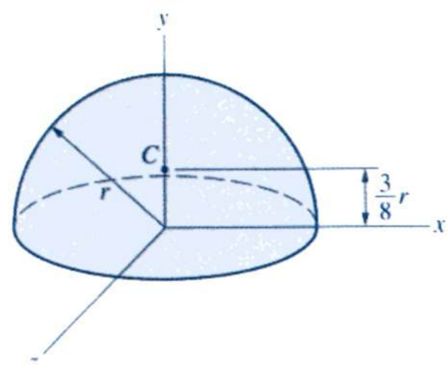
$V = \pi r^2 L$

Sphere



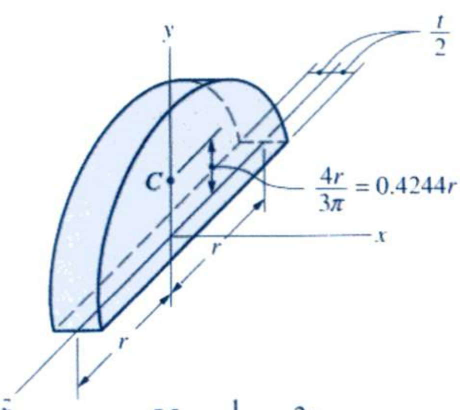
$V = \frac{4}{3}\pi r^3$

Hemisphere



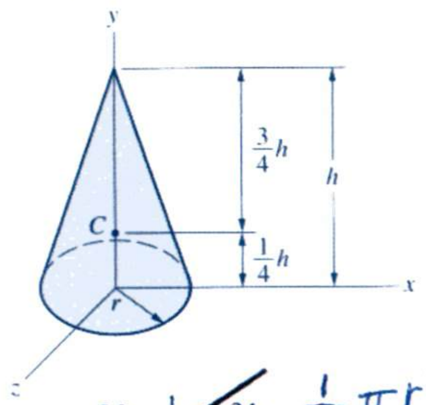
$V = \frac{2}{3}\pi r^3$

Semicircular Disk



$V = \frac{1}{2}\pi r^2 t$

Circular Cone



~~$V = \frac{1}{2}\pi r^2 h$~~ $\frac{1}{3}\pi r^2 h$

7–2 Locate the centroid of a body consisting of the cylinder and circular cone shown in Fig. P7–2.

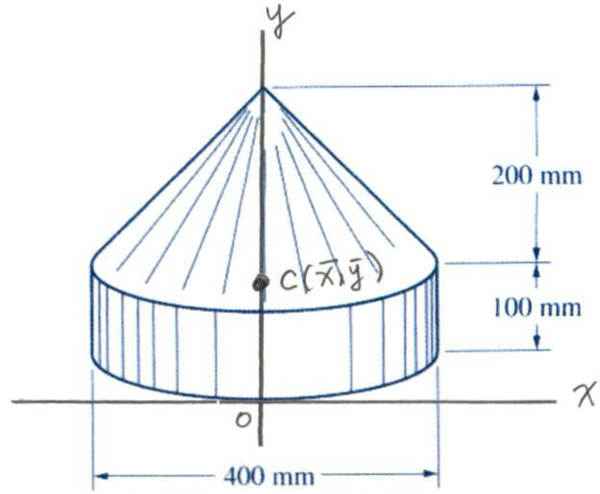


FIGURE P7–2

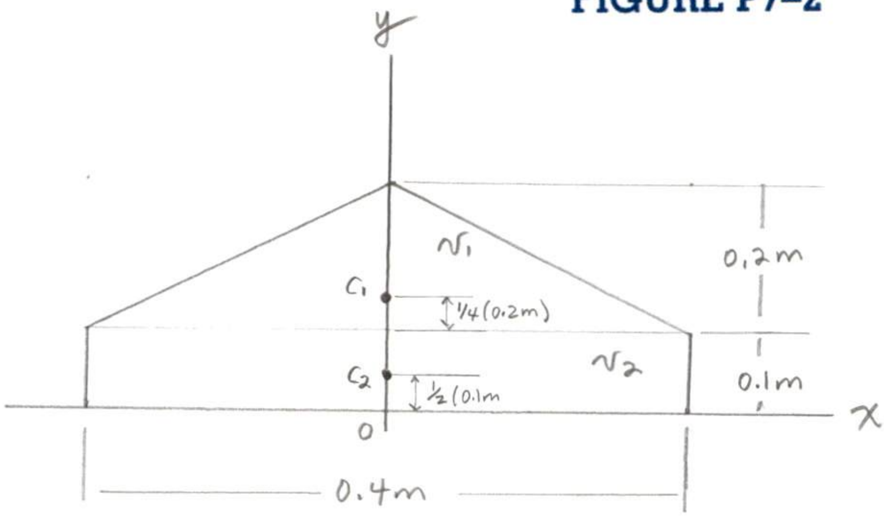
Solution.

Find

$C(\bar{x}, \bar{y})$

By Symmetry,

$\bar{x} = 0$



Circular Cone

$$v_1 = \frac{1}{3}\pi r^2 h = \frac{1}{3}\pi (0.2m)^2 (0.2m) = 0.00838\ m^3$$

Cylinder

$$v_2 = \pi r^2 L = \pi (0.2m)^2 (0.1m) = 0.01257\ m^3$$

$$\bar{y} = \frac{v_1 y_1 + v_2 y_2}{v_1 + v_2}$$

$$= \frac{0.00838m^3 \left(0.1m + \frac{1}{4}(0.2m)\right) + 0.01257m^3 \left(\frac{0.1m}{2}\right)}{0.00838\ m^3 + 0.01257m^3}$$

$= 0.09m$

$= 90\ mm$

C (0, 90mm)

7–4 The mallet in Fig. P7–4 has a head made of steel and a cylindrical handle made of wood. The handle fits tightly into a hole over the entire width of the head. Locate the center of gravity of the mallet given $\gamma_{st} = 490\ \text{lb/ft}^3$, and $\gamma_{wd} = 40\ \text{lb/ft}^3$.

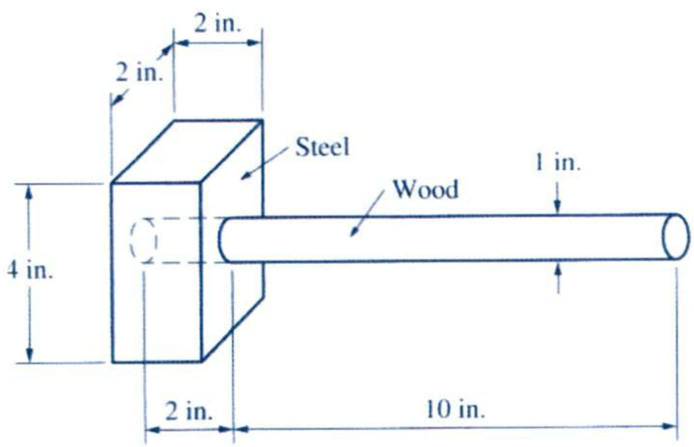
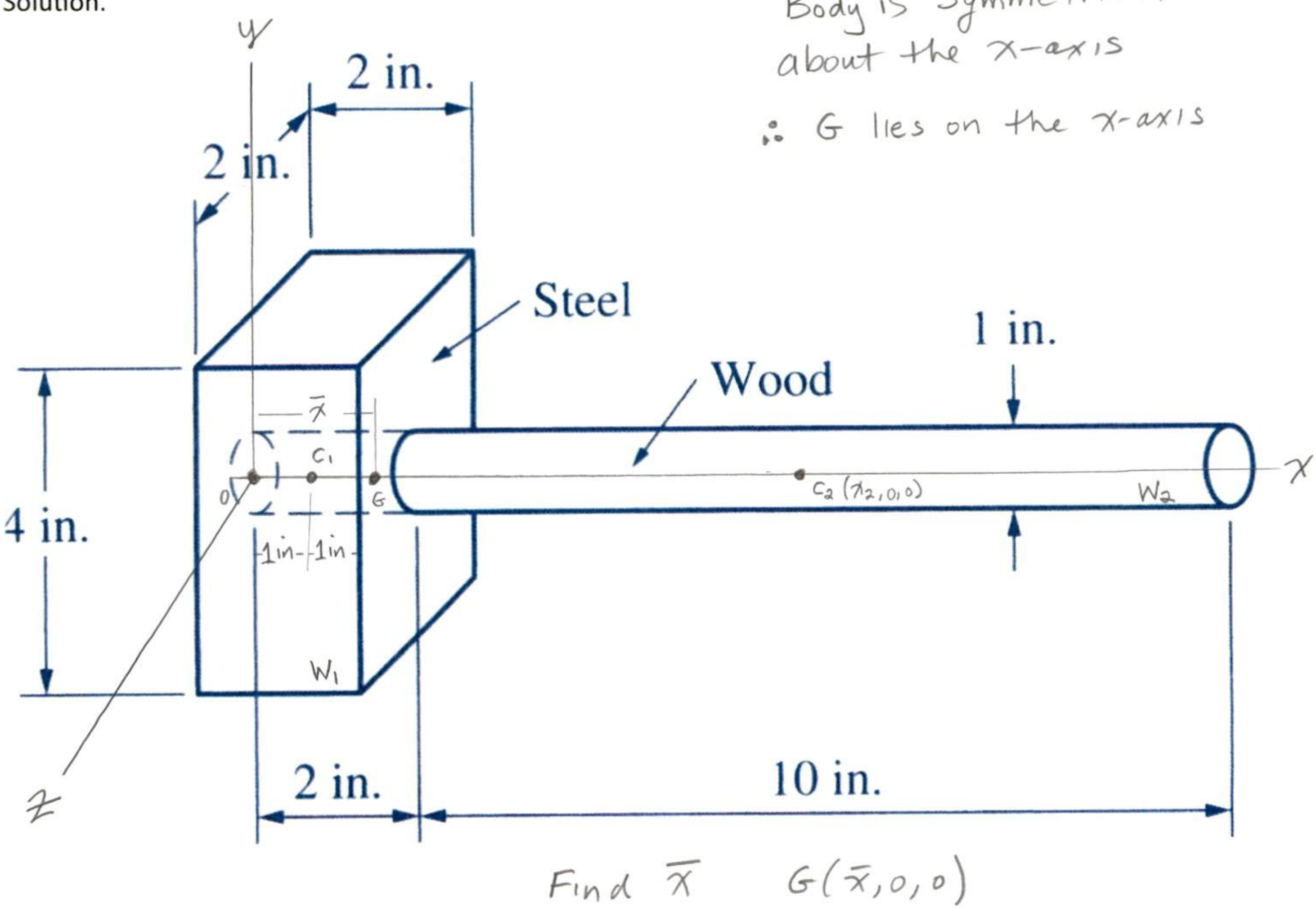


FIGURE P7–4

Solution.



The mallet consist of two component weights: the rectangular steel head (W_1) and the cylindrical wood handle (W_2)

$$\gamma_{ST} = \frac{490\text{ lb}}{ft^3} \qquad \gamma_{WD} = \frac{40\text{ lb}}{ft^3}$$

$$W_1 = V_1 \gamma_{ST}$$

$$= \frac{\left[(2in)(2in)(4in) - \pi(0.5in)^2(2in)\right] 1\,ft^3}{12\,in^3}\left(\frac{490\text{ lb}}{ft^3}\right)$$

$$= 4.09\text{ lb}$$

$$W_2 = V_2 \gamma_{WD}$$

$$= \frac{\left[\pi(0.5in)^2(12\,in)\right] 1\,ft^3}{12\,in^3}\left(\frac{40\text{ lb}}{ft^3}\right) = 0.218\text{ lb}$$

$$\bar{x} = \frac{W_1 x_1 + W_2 x_2}{W_1 + W_2}$$

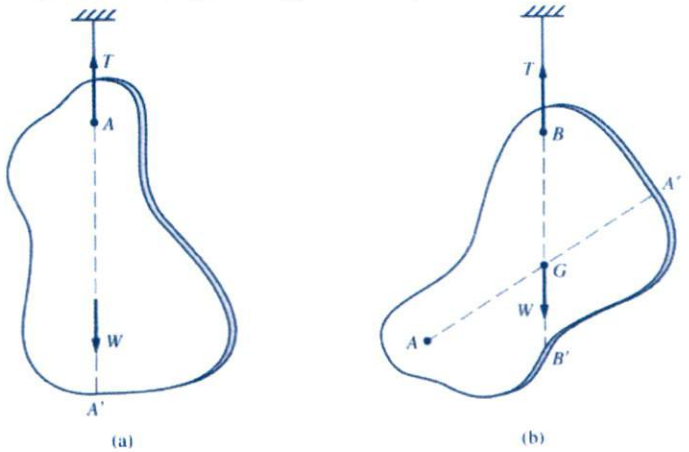
$$= \frac{4.09\text{ lb}(1in) + 0.218\text{ lb}(6in)}{4.09\text{ lb} + 0.218\text{ lb}}$$

$$= 1.25\text{ in}$$

$$\boxed{G(1.25, 0, 0)}$$

7-3
Experimental Determination of the Center of Gravity

Method 1. Suspending the body

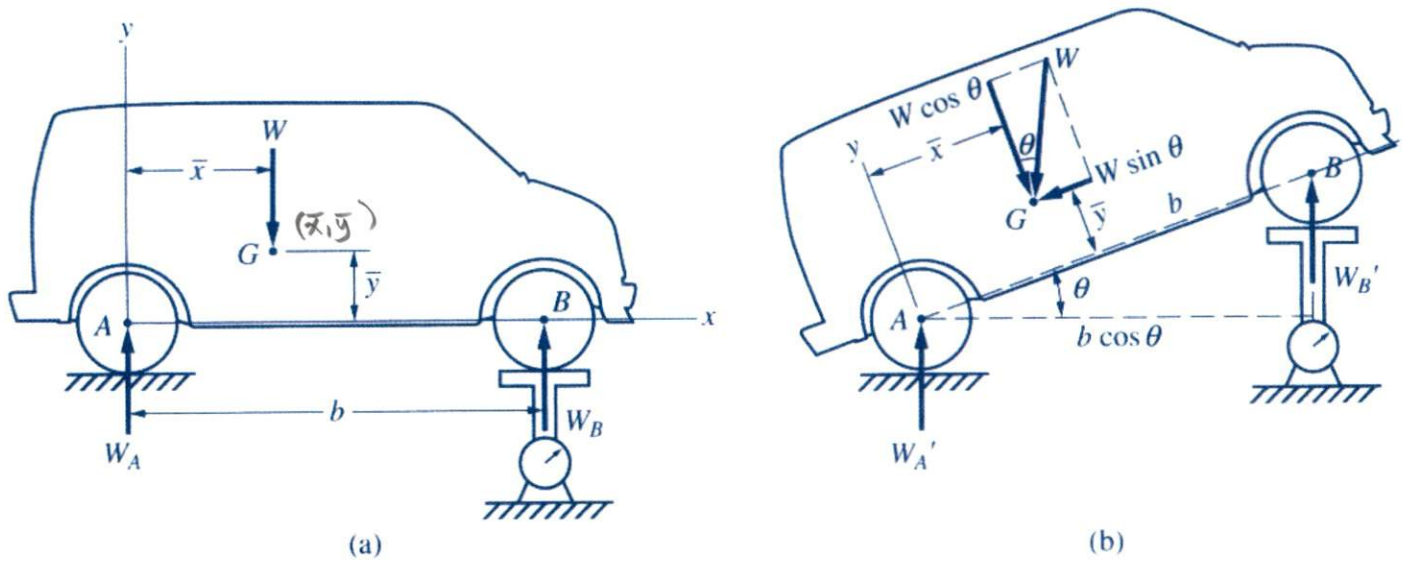


Method 2. Weighing Method

$$\Sigma M_A = -W\bar{x} + W_B b = 0$$

From which we get

$$\bar{x} = \frac{W_B}{W} b \qquad (7\text{–}4)$$



$$\Sigma M_A = (W \sin\theta)\bar{y} - (W \cos\theta)\bar{x} + W_B' b \cos\theta = 0$$

Substituting $\bar{x}$ from Equation 7–4 and solving for $\bar{y}$, we get

$$\bar{y} = \frac{W_B b \cos\theta - W_B' b \cos\theta}{W \sin\theta}$$

or

$$\bar{y} = \frac{b}{W \tan\theta}(W_B - W_B') \qquad (7\text{–}5)$$

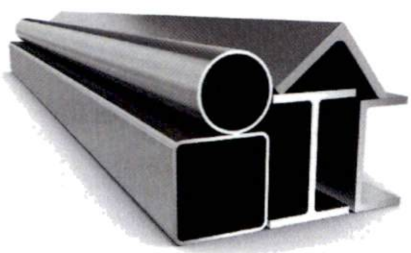
7-4
Centroid of an Area

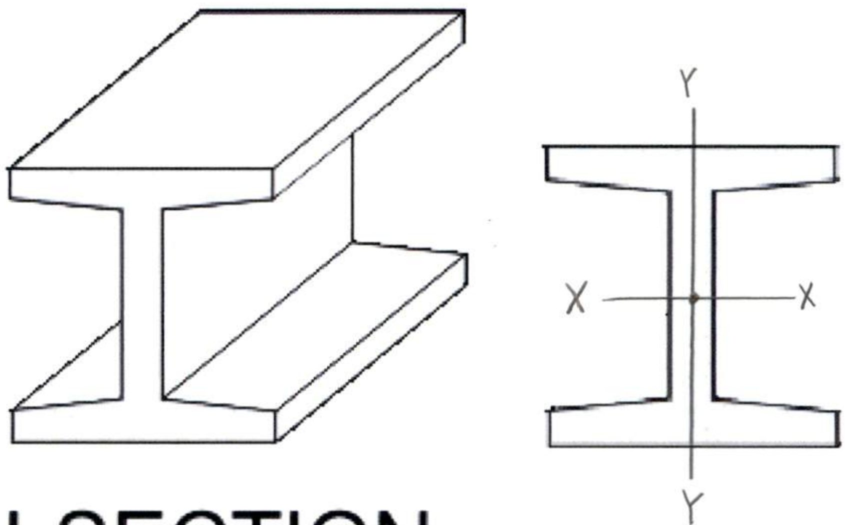
Centroid, C
Defines the geometric center of an object

Uniform Body – density is constant (γ)
also called, specific weight

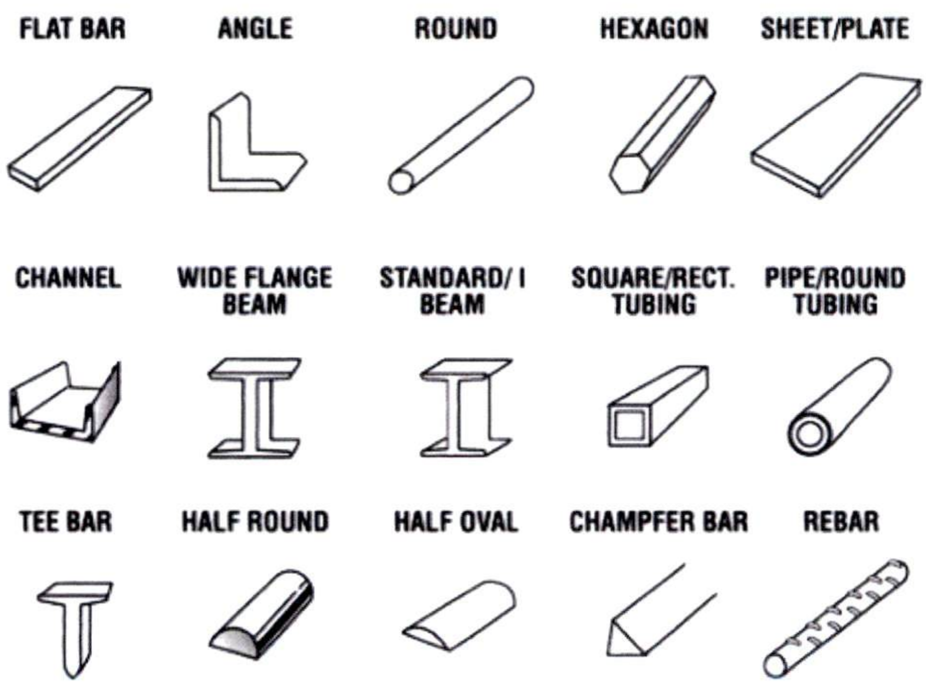
C, coincides with G, if the body is uniform.

Symmetry
If the shape (body) has an axis of symmetry, the centroid, C, of the shape will lie along that axis.



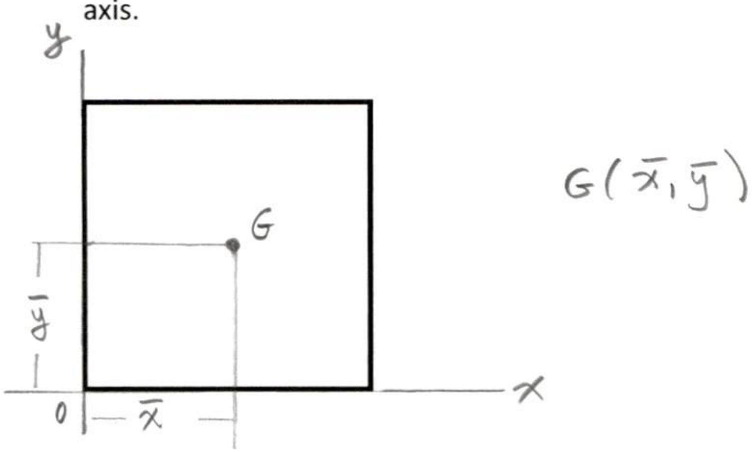


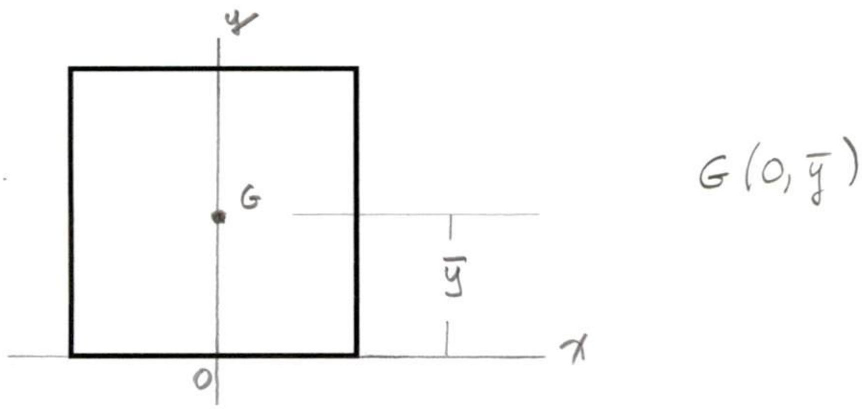
I-SECTION

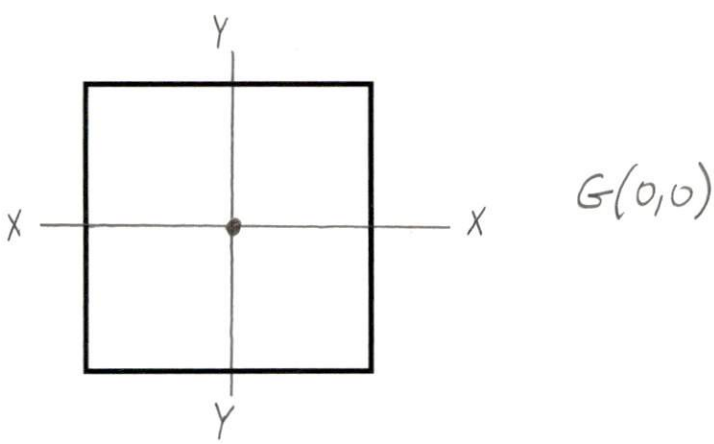


Centroid of an Area

Reference Axis – the location of the centroid of a shape is determined with respect to a given reference axis.







Centroid of a Composite Area

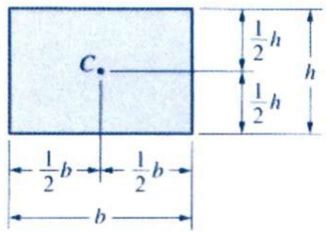
$$\bar{x} = \frac{\Sigma Ax}{\Sigma A} = \frac{A_1x_1 + A_2x_2 + A_3x_3 + A_4x_4}{A_1 + A_2 + A_3 + A_4}$$

(7–7)

$$\bar{y} = \frac{\Sigma Ay}{\Sigma A} = \frac{A_1y_1 + A_2y_2 + A_3y_3 + A_4y_4}{A_1 + A_2 + A_3 + A_4}$$

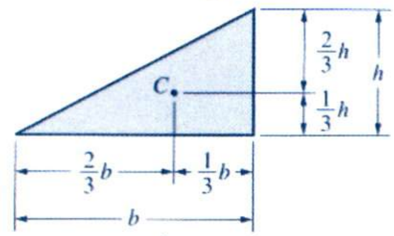
TABLE 7–2 Centroids of Areas of Common Shapes

Rectangle



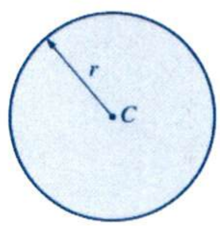
$A = bh$

Triangle



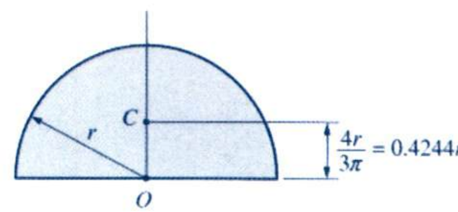
$A = \frac{1}{2}bh$

Circle



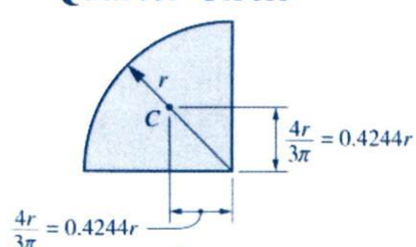
$A = \pi r^2$

Semicircle



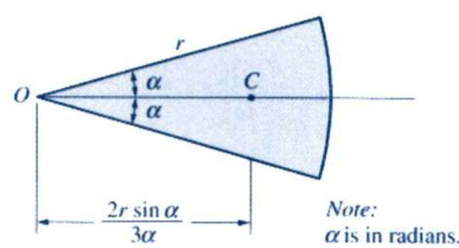
$A = \frac{1}{2}\pi r^2$

Quarter-Circle



$A = \frac{1}{4}\pi r^2$

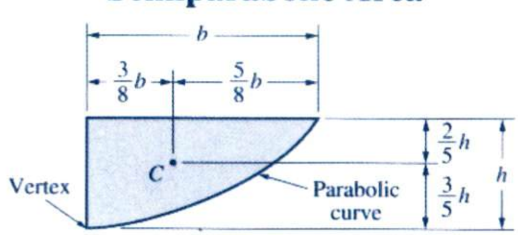
Sectors



Note:
α is in radians.

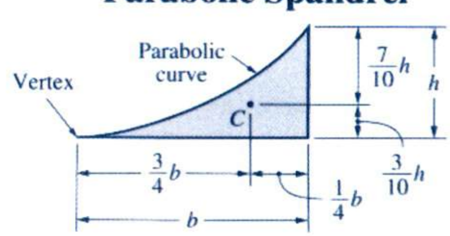
$A = \alpha r^2$

Semiparabolic Area



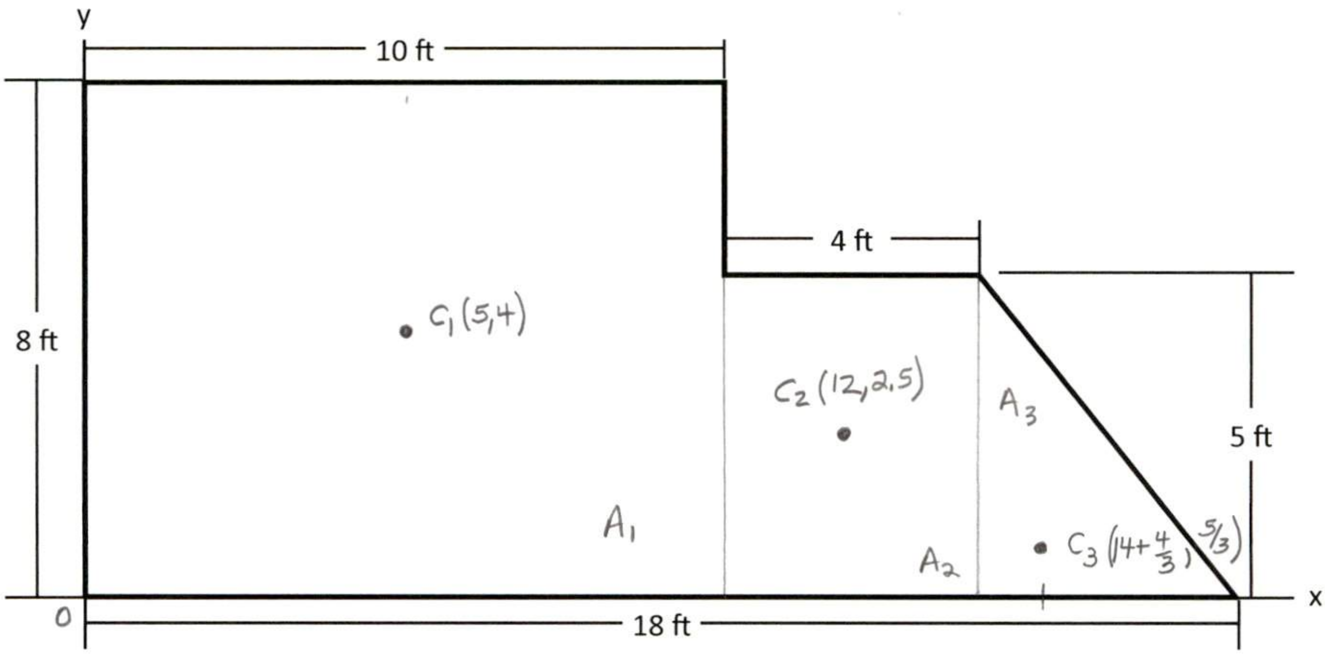
$A = \frac{2}{3}bh$

Parabolic Spandrel



$A = \frac{1}{3}bh$

Example 1. Locate the centroid of the plane area shown. Use the given reference axis.



Solution.

1. Break the composite shape into common geometric areas.
2. Choose a Reference Axis or use the given Reference Axis
3. Construct a table as shown below.

Shape	Area (ft²)	x (ft)	Ax (ft³)	y (ft)	Ay (ft³)
A_1	10x8 = 80	5	400	4	320
A_2	4x5 = 20	12	240	2.5	50
A_3	$\frac{1}{2}(4)(5) = 10$	$15\frac{1}{3}$	153.3	$5/3$	16.7
Σ	110		793.3		386.7

4. Find x̄ and ȳ

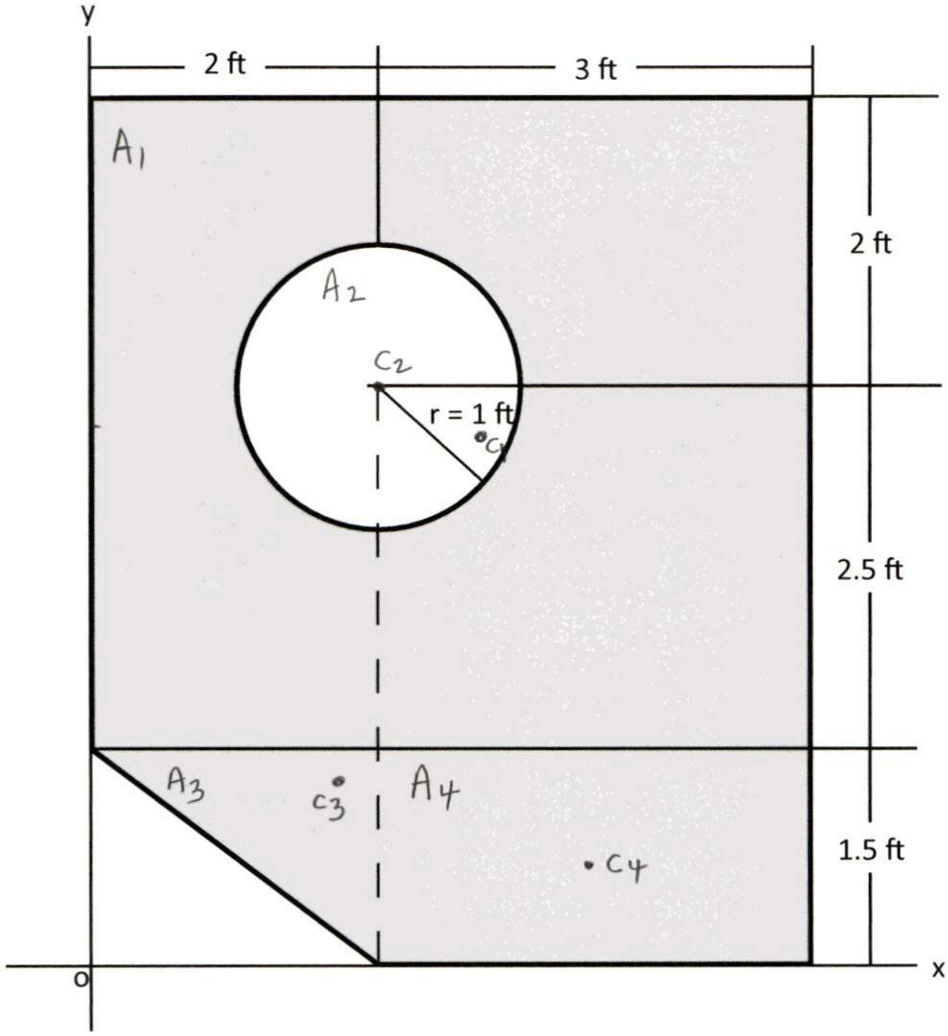
$$\bar{x} = \frac{\Sigma Ax}{\Sigma A} = \frac{793.3\ ft^3}{110\ ft^2} = 7.2\ ft$$

$$\bar{y} = \frac{\Sigma Ay}{\Sigma A} = \frac{386.7\ ft^3}{110\ ft^2} = 3.5\ ft$$

Location of the Centroid,

C (7.2 ft , 3.5 ft)

Example 2. Locate the centroid of the plane area shown. Use the given reference axis.



Solution.

Shape	Area (ft²)	x (ft)	Ax (ft³)	y (ft)	Ay (ft³)
A_1	$5 \times 4.5 = 22.5$	2.5	56.25	3.75	84.375
CUT-OUT A_2	$-\pi(1)^2 = -3.14$	2	−6.28	4	−12.56
A_3	$\frac{1}{2}(2)(1.5) = 1.5$	4/3	2.0	1.0	1.5
A_3	$3 \times 1.5 = 4.5$	3.5	15.75	0.75	3.375
Σ	25.36		67.72		76.69

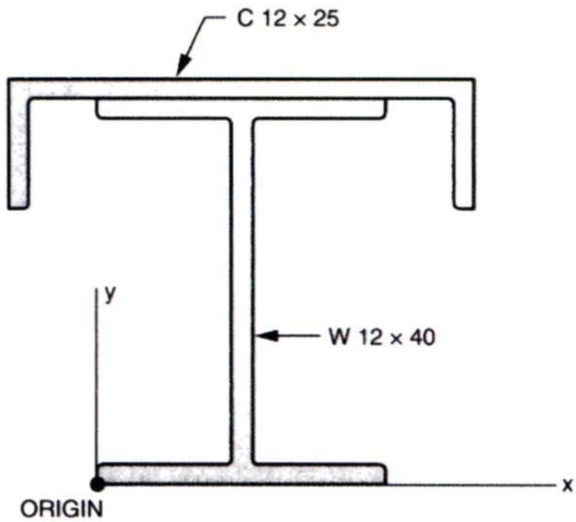
$$\bar{x} = \frac{\Sigma Ax}{\Sigma A} = \frac{67.72 \text{ ft}^3}{25.36 \text{ ft}^2} = 2.67 \text{ ft}$$

$$\bar{y} = \frac{\Sigma Ay}{\Sigma A} = \frac{76.69 \text{ ft}^3}{25.36 \text{ ft}^2} = 3.02 \text{ ft}$$

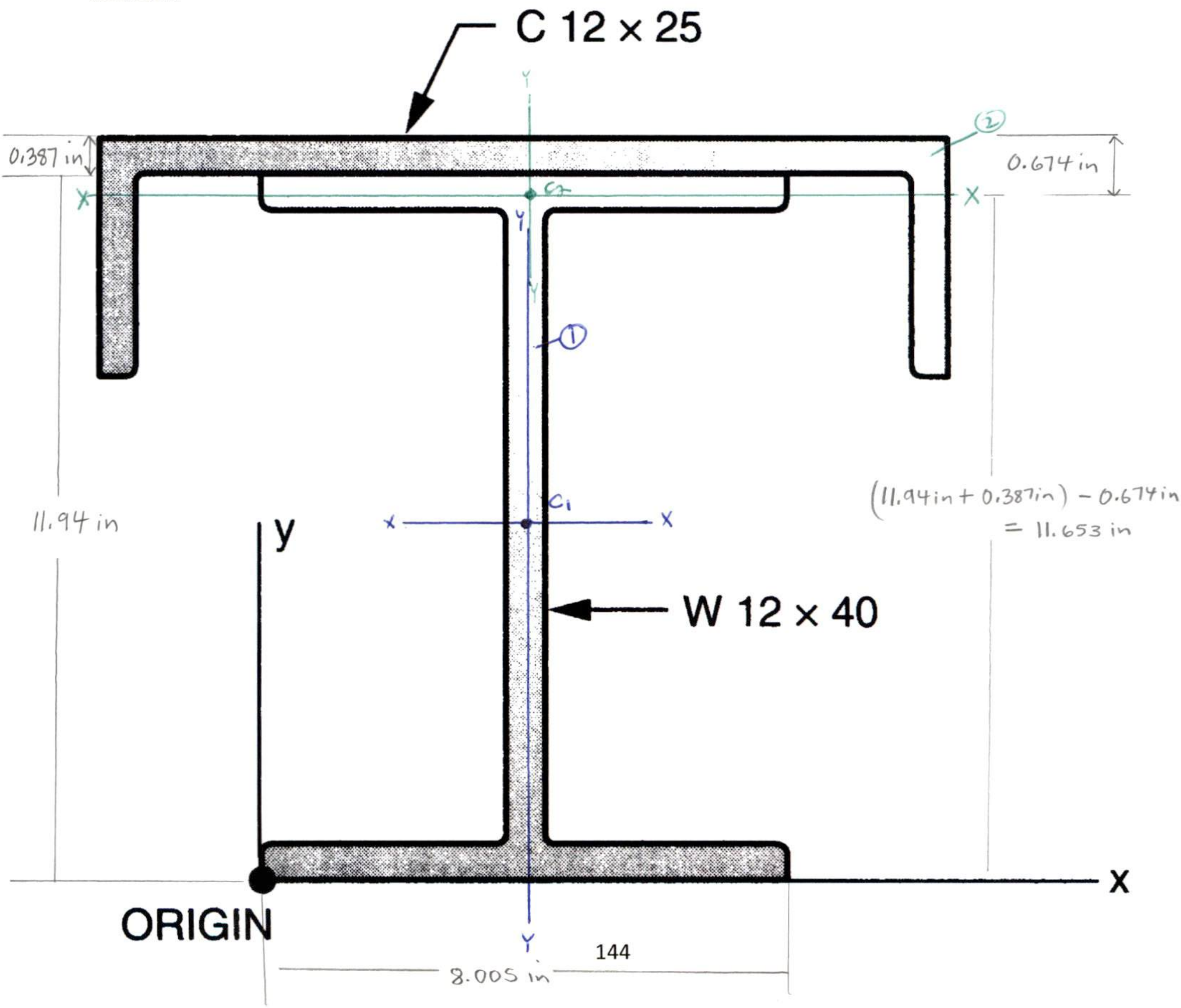
C (2.67 ft, 3.02 ft)

Example 3.

Locate the centroid of the built-up structural steel section with respect to the given reference axis.



Solution.



Structural Steel Properties can be found in Appendix A of the text book. The following tables can be found in the course notes on pages 146 and 147.

Table A-1(a) Properties of Selected W Shapes (Wide-Flange Sections)
Table A-3(a) Properties of C Shapes (Channels)

Shape	Area (in²)	x (ft)	Ax (in³)	y (ft)	Ay (in³)
W12x40	11.8	4.0025	47.2295	$\frac{11.94}{2}$ = 5.97	70.446
C12x25	7.35	4.0025	29.4184	12.327−0.674 = 11.653	85.0956
Σ	19.15		76.6479		156.0956

$$\bar{x} = \frac{76.6479 \text{ in}^3}{19.15 \text{ in}^2} = 4.0025 \text{ in}$$

$$\bar{y} = \frac{156.0956 \text{ in}^3}{19.15 \text{ in}^2} = 8.1512 \text{ in}$$

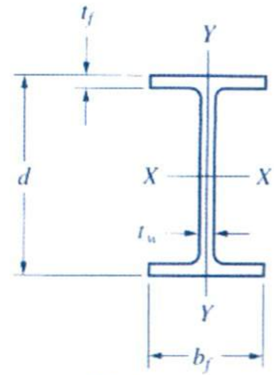


TABLE A–1(a) (Continued) **Properties of Selected W Shapes (Wide-Flange Sections): U.S. Customary Units**

Designation	Area	Depth	Web Thickness	Flange Width	Flange Thickness	Elastic Properties Axis x–x			Axis y–y			Plastic Modulus	
	A	d	t_w	b_f	t_f	I	S	r	I	S	r	Z_X	Z_Y
(in. × lb/ft)	(in.^{2})	(in.)	(in.)	(in.)	(in.)	(in.^{4})	(in.^{3})	(in.)	(in.^{4})	(in.^{3})	(in.)	(in.^{3})	(in.^{3})
W14 × 74	21.8	14.17	0.450	10.070	0.785	796	112	6.04	134	26.6	2.48	126	40.6
× 68	20.0	14.04	0.415	10.035	0.720	723	103	6.01	121	24.2	2.46	115	36.9
× 61	17.9	13.89	0.375	9.995	0.645	640	92.2	5.98	107	21.5	2.45	102	32.8
× 53	15.6	13.92	0.370	8.060	0.660	541	77.8	5.89	57.7	14.3	1.92	87.1	22.0
× 43	12.6	13.66	0.305	7.995	0.530	428	62.7	5.82	45.2	11.3	1.89	69.6	17.3
× 38	11.2	14.10	0.310	6.770	0.515	385	54.6	5.87	26.7	7.88	1.55	61.5	12.1
× 34	10.0	13.98	0.285	6.745	0.455	340	48.6	5.83	23.3	6.91	1.53	54.6	10.6
× 30	8.85	13.84	0.270	6.730	0.385	291	42.0	5.73	19.6	5.82	1.49	47.3	8.99
W12 × 87	25.6	12.53	0.515	12.125	0.810	740	118	5.38	241	39.7	3.07	132	60.4
× 65	19.1	12.12	0.390	12.000	0.605	533	87.9	5.28	174	29.1	3.02	96.8	44.1
× 53	15.6	12.06	0.345	9.995	0.575	425	70.6	5.23	95.8	19.2	2.48	77.9	29.1
× 40	11.8	11.94	0.295	8.005	0.515	310	51.9	5.13	44.1	11.0	1.93	57.5	16.8
× 35	10.3	12.50	0.300	6.560	0.520	285	45.6	5.25	24.5	7.47	1.54	51.2	11.5
× 30	8.79	12.34	0.260	6.520	0.440	238	38.6	5.21	20.3	6.24	1.52	43.1	9.56
× 22	6.48	12.31	0.260	4.030	0.425	156	25.4	4.91	4.66	2.31	0.847	29.3	3.66
W10 × 112	32.9	11.36	0.755	10.415	1.250	716	126	4.66	236	45.3	2.68	147	69.2
× 100	29.4	11.10	0.680	10.340	1.120	623	112	4.60	207	40.0	2.65	130	61.0
× 88	25.9	10.84	0.605	10.265	0.990	534	98.5	4.54	179	34.8	2.63	113	53.1
× 77	22.6	10.60	0.530	10.190	0.870	455	85.9	4.49	154	30.1	2.60	97.6	45.9
× 60	17.6	10.22	0.420	10.080	0.680	341	66.7	4.39	116	23.0	2.57	74.6	35.0
× 49	14.4	9.98	0.340	10.000	0.560	272	54.6	4.35	93.4	18.7	2.54	60.4	28.3
× 45	13.3	10.10	0.350	8.020	0.620	248	49.1	4.32	53.4	13.3	2.01	54.9	20.3
× 39	11.5	9.92	0.315	7.985	0.530	209	42.1	4.27	45.0	11.3	1.98	46.8	17.2
× 33	9.71	9.73	0.290	7.960	0.435	170	35.0	4.19	36.6	9.20	1.94	38.8	14.0
× 22	6.49	10.17	0.240	5.750	0.360	118	23.2	4.27	11.4	3.97	1.33	26.0	6.10
W8 × 67	19.7	9.00	0.570	8.280	0.935	272	60.4	3.72	88.6	21.4	2.12	70.2	32.7
× 58	17.1	8.75	0.510	8.220	0.810	228	52.0	3.65	75.1	18.3	2.10	59.8	27.9
× 48	14.1	8.50	0.400	8.110	0.685	184	43.3	3.61	60.9	15.0	2.08	49.0	22.9
× 40	11.7	8.25	0.360	8.070	0.560	146	35.5	3.53	49.1	12.2	2.04	39.8	18.5
× 35	10.3	8.12	0.310	8.020	0.495	127	31.2	3.51	42.6	10.6	2.03	34.7	16.1
× 31	9.13	8.00	0.285	7.995	0.435	110	27.5	3.47	37.1	9.27	2.02	30.4	14.1
× 28	8.25	8.06	0.285	6.535	0.465	98.0	24.3	3.45	21.7	6.63	1.62	27.2	10.1
× 24	7.08	7.93	0.245	6.495	0.400	82.8	20.9	3.42	18.3	5.63	1.61	23.2	8.57
× 21	6.16	8.28	0.250	5.270	0.400	75.3	18.2	3.49	9.77	3.71	1.26	20.4	5.69
× 18	5.26	8.14	0.230	5.250	0.330	61.9	15.2	3.43	7.97	3.04	1.23	17.0	4.66

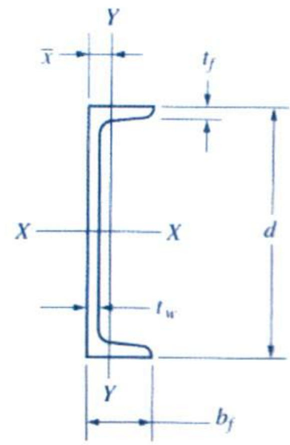
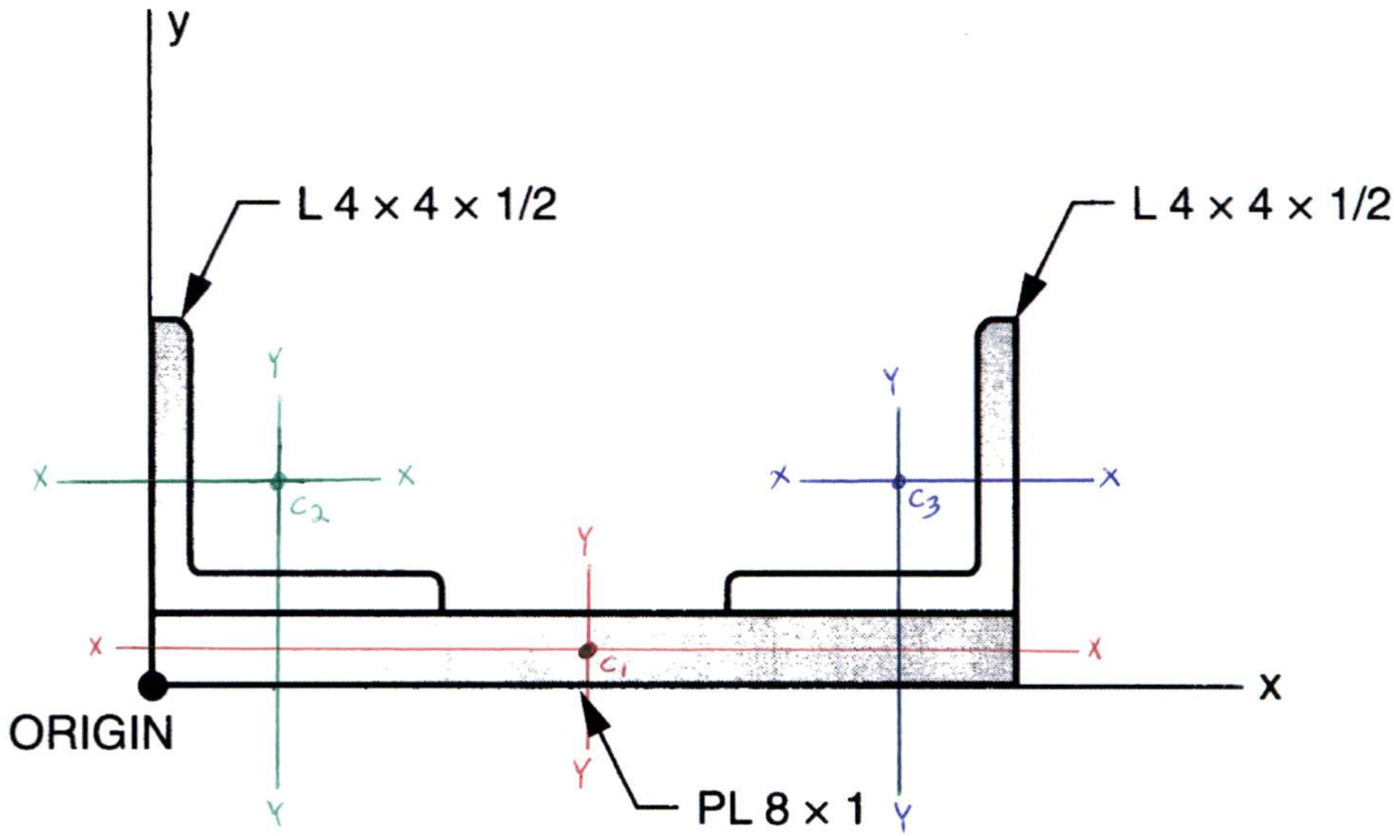


TABLE A–3(a) Properties of C Shapes (American Standard Channels): U.S. Customary Units

Designation	Area	Depth	Web	Flange		Axis x–x			Axis y–y			Centroid
			Thickness	Width	Average Thickness							
in. × lb/ft	A (in.²)	d (in.)	t_w (in.)	b_f (in.)	t_f (in.)	I (in.⁴)	S (in.³)	r (in.)	I (in.⁴)	S (in.³)	r (in.)	$\bar{x}$ (in.)
C15 × 50	14.7	15.00	0.716	3.716	0.650	404	53.8	5.24	11.0	3.78	0.867	0.798
× 40	11.8	15.00	0.520	3.520	0.650	349	46.5	5.44	9.23	3.37	0.886	0.777
× 33.9	9.96	15.00	0.400	3.400	0.650	315	42.0	5.62	8.13	3.11	0.904	0.787
C12 × 30	8.82	12.00	0.510	3.170	0.501	162	27.0	4.29	5.14	2.06	0.763	0.674
× 25	7.35	12.00	0.387	3.047	0.501	144	24.1	4.43	4.47	1.88	0.780	0.674
× 20.7	6.09	12.00	0.282	2.942	0.501	129	21.5	4.61	3.88	1.73	0.799	0.698
C10 × 30	8.82	10.00	0.673	3.033	0.436	103	20.7	3.42	3.94	1.65	0.669	0.649
× 25	7.35	10.00	0.526	2.886	0.436	91.2	18.2	3.52	3.36	1.48	0.676	0.617
× 20	5.88	10.00	0.379	2.739	0.436	78.9	15.8	3.66	2.81	1.32	0.692	0.606
× 15.3	4.49	10.00	0.240	2.600	0.436	67.4	13.5	3.87	2.28	1.16	0.713	0.634
C 9 × 20	5.88	9.00	0.448	2.648	0.413	60.9	13.5	3.22	2.42	1.17	0.642	0.583
× 15	4.41	9.00	0.285	2.485	0.413	51.0	11.3	3.40	1.93	1.01	0.661	0.586
× 13.4	3.94	9.00	0.233	2.433	0.413	47.9	10.6	3.48	1.76	0.962	0.669	0.601
C 8 × 18.75	5.51	8.00	0.487	2.527	0.390	44.0	11.0	2.82	1.98	1.01	0.599	0.565
× 13.75	4.04	8.00	0.303	2.343	0.390	36.1	9.03	2.99	1.53	0.854	0.615	0.553
× 11.5	3.38	8.00	0.220	2.260	0.390	32.6	8.14	3.11	1.32	0.781	0.625	0.571
C 7 × 14.75	4.33	7.00	0.419	2.299	0.366	27.2	7.78	2.51	1.38	0.779	0.564	0.532
× 12.25	3.60	7.00	0.314	2.194	0.366	24.2	6.93	2.60	1.17	0.703	0.571	0.525
× 9.8	2.87	7.00	0.210	2.090	0.366	21.3	6.08	2.72	0.968	0.625	0.581	0.540
C 6 × 13	3.83	6.00	0.437	2.157	0.343	17.4	5.80	2.13	1.05	0.642	0.525	0.514
× 10.5	3.09	6.00	0.314	2.034	0.343	15.2	5.06	2.22	0.866	0.564	0.529	0.499
× 8.2	2.40	6.00	0.200	1.920	0.343	13.1	4.38	2.34	0.693	0.492	0.537	0.511
C 5 × 9	2.64	5.00	0.325	1.885	0.320	8.96	3.56	1.83	0.632	0.450	0.489	0.478
× 6.7	1.97	5.00	0.190	1.750	0.320	7.49	3.00	1.95	0.479	0.378	0.493	0.484
C 4 × 7.25	2.13	4.00	0.321	1.721	0.296	4.59	2.29	1.47	0.433	0.343	0.450	0.459
× 5.4	1.59	4.00	0.184	1.584	0.296	3.85	1.93	1.56	0.319	0.283	0.449	0.457
C 3 × 6	1.76	3.00	0.356	1.596	0.273	2.07	1.38	1.08	0.305	0.268	0.416	0.455
× 5	1.47	3.00	0.258	1.498	0.273	1.85	1.24	1.12	0.247	0.233	0.410	0.438
× 4.1	1.21	3.00	0.170	1.410	0.273	1.66	1.10	1.17	0.197	0.202	0.404	0.436

Example 4.

Locate the centroid of the built-up structural steel section with respect to the given reference axis.



Solution.

Shape	Area (in²)	x (ft)	Ax (in³)	y (ft)	Ay (in³)
PL 8x1	8	4	32	1/2	4
L 4x4x 1/2	3.75	1.18	4.425	1+1.18 = 2.18	8.175
L 4x4x 1/2	3.75	8−1.18 = 6.82	25.575	2.18	8.175
Σ	15.5		62		20.35

$$\bar{x} = \frac{62\ in^3}{15.5\ in^2} = 4\ in$$

$$\bar{y} = \frac{20.35\ in^3}{15.5\ in^2} = 1.31\ in$$

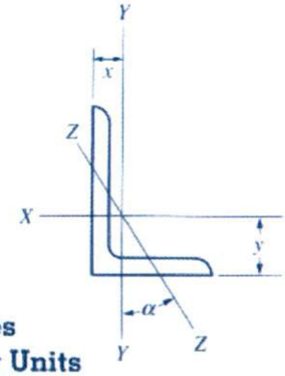
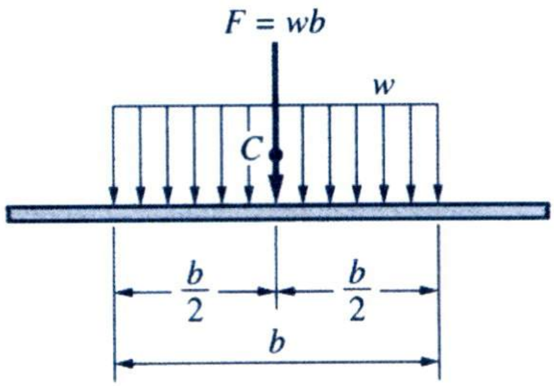


TABLE A–4(a) (Continued) **Properties of Selected L Shapes (Steel Angles): U.S. Customary Units**

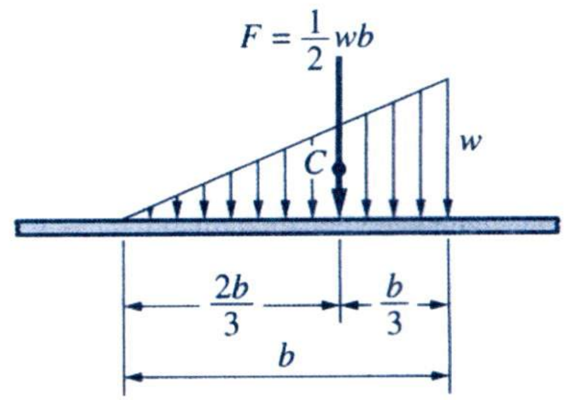
Size and Thickness		Weight per ft	Area	Axis x–x				Axis y–y				Axis z–z	
			A	I	S	r	y	I	S	r	x	r	tan α
(in.)		(lb/ft)	(in.^{2})	(in.^{4})	(in.^{3})	(in.)	(in.)	(in.^{4})	(in.^{3})	(in.)	(in.)	(in.)	
L 4 × 4	× $\frac{3}{4}$	18.5	5.44	7.67	2.81	1.19	1.27	7.67	2.81	1.19	1.27	0.778	1.000
	× $\frac{5}{8}$	15.7	4.61	6.66	2.40	1.20	1.23	6.66	2.40	1.20	1.23	0.779	1.000
	× $\frac{1}{2}$	12.8	3.75	5.56	1.97	1.22	1.18	5.56	1.97	1.22	1.18	0.782	1.000
	× $\frac{3}{8}$	9.8	2.86	4.36	1.52	1.23	1.14	4.36	1.52	1.23	1.14	0.788	1.000
	× $\frac{5}{16}$	8.2	2.40	3.71	1.29	1.24	1.12	3.71	1.29	1.24	1.12	0.791	1.000
	× $\frac{1}{4}$	6.6	1.94	3.04	1.05	1.25	1.09	3.04	1.05	1.25	1.09	0.795	1.000
L 4 × 3$\frac{1}{2}$	× $\frac{1}{2}$	11.9	3.50	5.32	1.94	1.23	1.25	3.79	1.52	1.04	1.00	0.722	0.750
	× $\frac{3}{8}$	9.1	2.67	4.18	1.49	1.25	1.21	2.95	1.17	1.06	0.955	0.727	0.755
	× $\frac{5}{16}$	7.7	2.25	3.56	1.26	1.26	1.18	2.55	0.994	1.07	0.932	0.730	0.757
	× $\frac{1}{4}$	6.2	1.81	2.91	1.03	1.27	1.16	2.09	0.808	1.07	0.909	0.734	0.579
L 4 × 3	× $\frac{1}{2}$	11.1	3.25	5.05	1.89	1.25	1.33	2.42	1.12	0.864	0.827	0.639	0.543
	× $\frac{3}{8}$	8.5	2.48	3.96	1.46	1.26	1.28	1.92	0.866	0.879	0.782	0.644	0.551
	× $\frac{5}{16}$	7.2	2.09	3.38	1.23	1.27	1.26	1.65	0.734	0.887	0.759	0.647	0.554
	× $\frac{1}{4}$	5.8	1.69	2.77	1.00	1.28	1.24	1.36	0.599	0.896	0.736	0.651	0.558
L 3$\frac{1}{2}$ × 3$\frac{1}{2}$	× $\frac{3}{8}$	8.5	2.48	2.87	1.15	1.07	1.01	2.87	1.15	1.07	1.01	0.687	1.000
	× $\frac{5}{16}$	7.2	2.09	2.45	0.976	1.08	0.990	2.45	0.976	1.08	0.990	0.690	1.000
	× $\frac{1}{4}$	5.8	1.69	2.01	0.794	1.09	0.968	2.01	0.794	1.09	0.968	0.694	1.000
L 3$\frac{1}{2}$ × 3	× $\frac{3}{8}$	7.9	2.30	2.72	1.13	1.09	1.08	1.85	0.851	0.897	0.830	0.625	0.721
	× $\frac{5}{16}$	6.6	1.93	2.33	0.954	1.10	1.06	1.58	0.722	0.905	0.808	0.627	0.724
	× $\frac{1}{4}$	5.4	1.56	1.91	0.776	1.11	1.04	1.30	0.589	0.914	0.785	0.631	0.727
L 3$\frac{1}{2}$ × 2$\frac{1}{2}$	× $\frac{3}{8}$	7.2	2.11	2.56	1.09	1.10	1.16	1.09	0.592	0.719	0.660	0.537	0.496
	× $\frac{5}{16}$	6.1	1.78	2.19	0.927	1.11	1.14	0.939	0.504	0.727	0.637	0.540	0.501
	× $\frac{1}{4}$	4.9	1.44	1.80	0.755	1.12	1.11	0.777	0.412	0.735	0.614	0.544	0.506
L 3 × 3	× $\frac{1}{2}$	9.4	2.75	2.22	1.07	0.898	0.932	2.22	1.07	0.898	0.932	0.584	1.000
	× $\frac{3}{8}$	7.2	2.11	1.76	0.833	0.913	0.888	1.76	0.833	0.913	0.888	0.587	1.000
	× $\frac{5}{16}$	6.1	1.78	1.51	0.707	0.922	0.865	1.51	0.707	0.922	0.865	0.589	1.000
	× $\frac{1}{4}$	4.9	1.44	1.24	0.577	0.930	0.842	1.24	0.577	0.930	0.842	0.592	1.000
	× $\frac{3}{16}$	3.71	1.09	0.962	0.441	0.939	0.820	0.962	0.441	0.939	0.820	0.596	1.000
L 3 × 2$\frac{1}{2}$	× $\frac{3}{8}$	6.6	1.92	1.66	0.810	0.928	0.956	1.04	0.581	0.736	0.706	0.522	0.676
	× $\frac{1}{4}$	4.5	1.31	1.17	0.561	0.945	0.911	0.743	0.404	0.753	0.661	0.528	0.684
	× $\frac{3}{16}$	3.39	0.996	0.907	0.430	0.954	0.888	0.577	0.310	0.761	0.638	0.533	0.688
L 3 × 2	× $\frac{3}{8}$	5.9	1.73	1.53	0.781	0.940	1.04	0.543	0.371	0.559	0.539	0.430	0.428
	× $\frac{5}{16}$	5.0	1.46	1.32	0.664	0.948	1.02	0.470	0.317	0.567	0.516	0.432	0.435
	× $\frac{1}{4}$	4.1	1.19	1.09	0.542	0.957	0.993	0.392	0.260	0.574	0.493	0.435	0.440
	× $\frac{3}{16}$	3.07	0.902	0.842	0.415	0.966	0.970	0.307	0.200	0.583	0.470	0.439	0.446

7-5
General Types of Distributed Line Loads

Distributed Load
A distributed load occurs whenever the load applied to a body is not concentrated at a point.



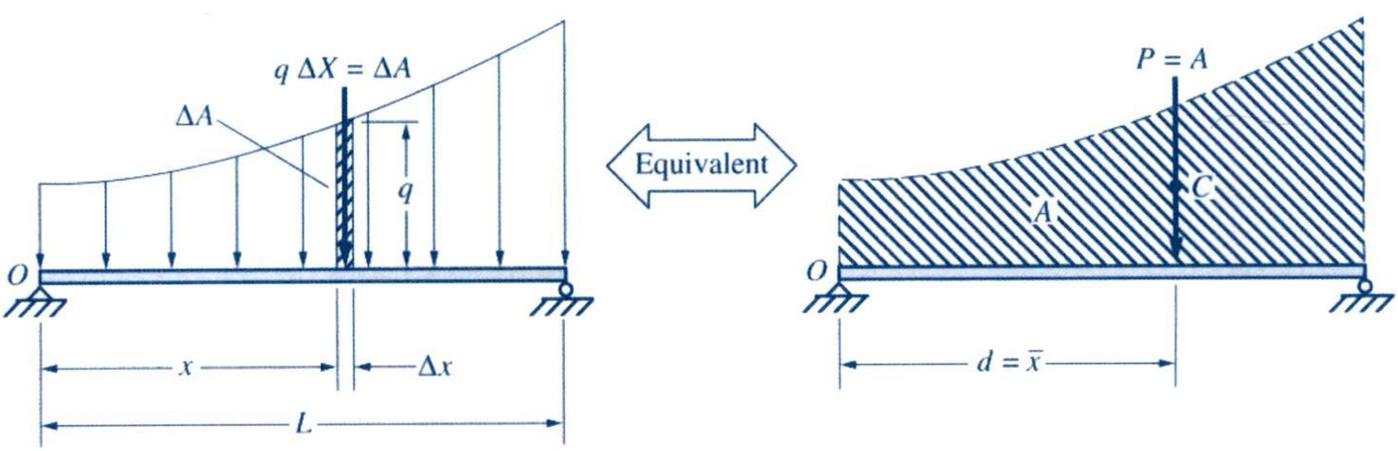
Uniform Distributed Load



Triangular Distributed Load

General Case

$$P = \Sigma q \Delta x$$



(a) Distributed load

(b) Equivalent resultant

q = load intensity
Δx = incremental length

$P = \Sigma\, q\Delta x$

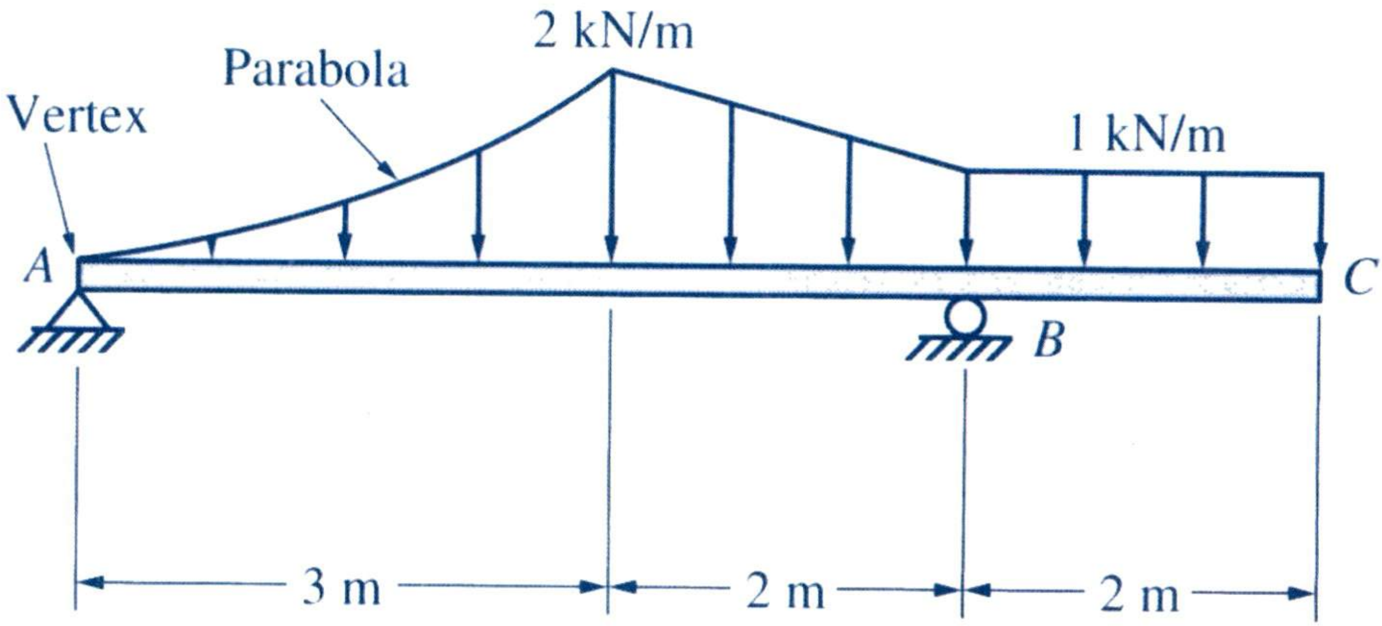
The total resultant over the entire length of the beam is: $P = \Sigma\, q\Delta x = \Sigma\, \Delta A = A$
Thus, the resultant of a distributed line load is equal to the area of the loading diagram.

The location of the resultant P along the beam may be determined by requiring that the moment of P about O be equal to the sum of the moments of the loads q Δx over the entire beam about O.

From the Equivalent Resultant diagram: $\Sigma\, M_O = P\, d = \Sigma\, (q\Delta x)x = \Sigma\, (\Delta A)x = A\,\bar{x} = P\,\bar{x}$

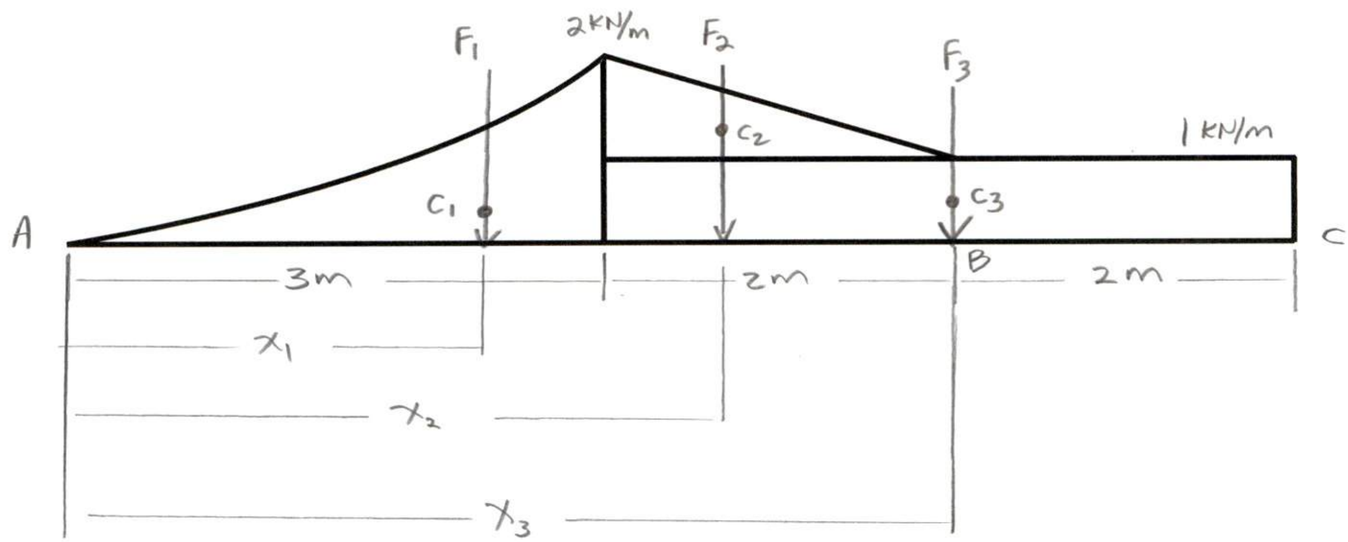
Thus,
$Pd = P\,\bar{x}$
$d = \bar{x}$ Line of action of the resultant P passes through the centroid of the area

Example 1. Determine the reactions at the supports of the beams for the loading shown.



Solution.

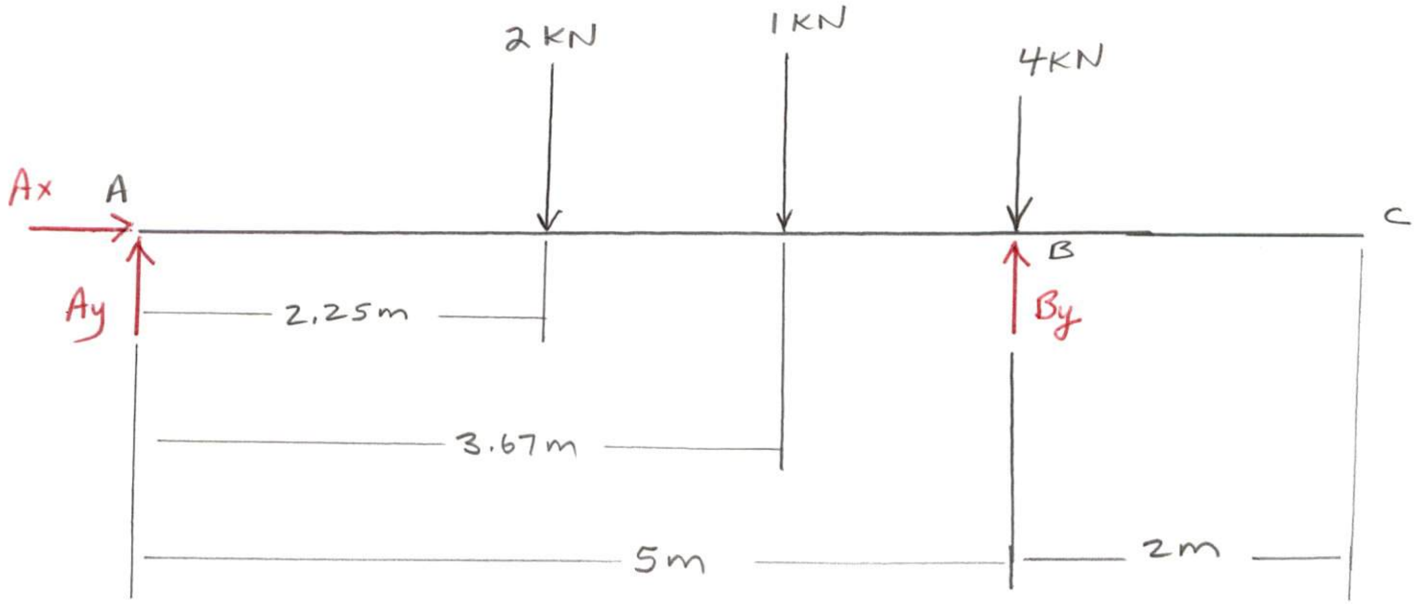
The loading diagram is divided into a parabolic spandrel, rectangular area, and triangular area.



$F_1 = \frac{1}{3}\left(2\,\frac{kN}{m}\right)(3\,m) = 2\,kN$ $\qquad x_1 = \frac{3}{4}(3) = 2.25\,m$

$F_2 = \frac{1}{2}\left(2\,\frac{kN}{m} - 1\,\frac{kN}{m}\right)(2\,m) = 1\,kN$ $\qquad x_2 = 3\,m + \frac{1}{3}(2\,m) = 3.67\,m$

$F_3 = 1\,\frac{kN}{m}(4\,m) = 4\,kN$ $\qquad x_3 = 3\,m + 2\,m = 5\,m$



FBD

CCW +M ↺
CW −M ↻

Equilibrium Equations

$[\Sigma F_x = 0]$ $\quad A_x = 0$

$[\Sigma M_A = 0]$ $\quad -2\,kN(2.25m) - 1\,kN(3.67m) - 4\,kN(5m) + B_y(5m) = 0$

$$B_y = \frac{28.17\,kN \cdot m}{5m} = \underline{\underline{5.63\ kN}} \uparrow$$

$[\Sigma F_y = 0]$ $\quad A_y - 2\,kN - 1\,kN - 4\,kN + B_y = 0$

$$A_y = 7\,kN - 5.63\,kN = \underline{\underline{1.37\ kN}} \uparrow$$