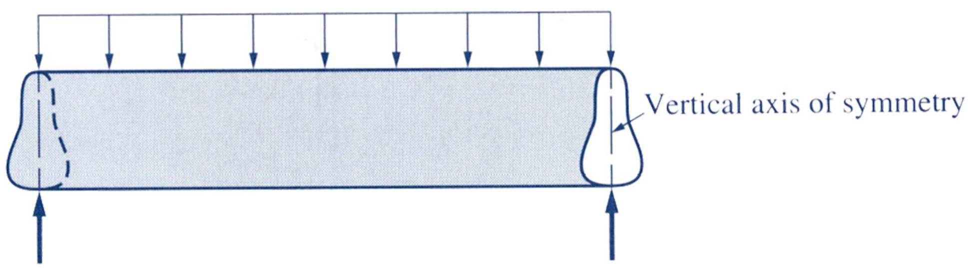
Chapter 13 - Shear Forces and Bending Moments in Beams
Reading: Chapter 13 — Pages 449-475

13-1
Introduction

- Many structures can be approximated as a straight beam or as a collection of straight beams. For this reason, the analysis of stresses and deflections in a beam is an important and useful topic.
- This section covers shear force and bending moment in beams, shear and moment diagrams, stresses in beams, and a table of common beam deflection formulas.

Beam Assumptions

1. Straight and of uniform cross-section, and that possess a vertical plane of symmetry, as shown below.
2. Horizontal, although in actual situations beams may be inclined or in vertical positions.
3. Subjected to forces applied in the vertical plane of symmetry, as shown below.

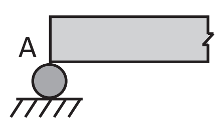


13-2
Types of Beams

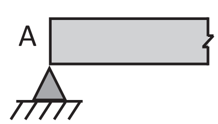
Types of Beam Support

Three main types of supports and Reactions

Roller



Pin (Hinge)

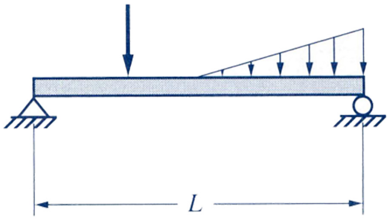


Fixed

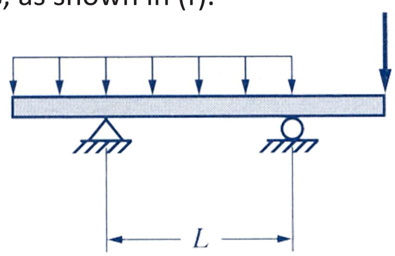
Types of Beams

Beams can be classified into the types shown below, according to the kind of support used.

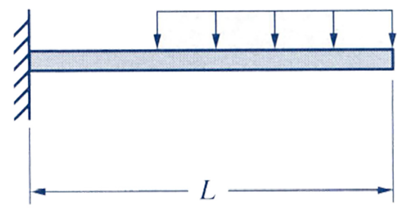
Simple Beam	A beam supported at its ends with a hinge and a roller, as shown in (a), is called a simple beam.
Overhanging Beam	A simply supported beam with an overhang from one or both ends, as shown in (b) is called an overhanging beam.
Cantilever Beam	A beam that is fixed at one end and free at the other, as shown in (c), is called a cantilever beam.
Propped Cantilever Beam	A beam that is fixed at one end and simply supported at the other, as shown in (d), is called a propped cantilever beam.
Fixed Beam	When both ends of a beam are fixed to supports, as shown in (e), the beam is called a fixed beam.
Continuous Beam	A continuous beam is supported on a hinge support and two or more roller supports, as shown in (f).



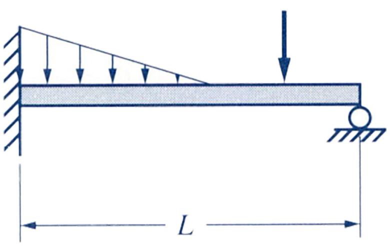
(a) Simple beam



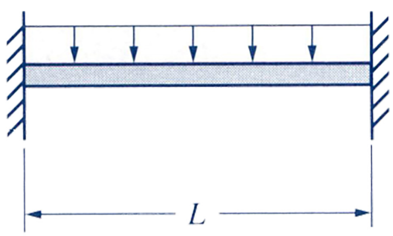
(b) Overhanging beam



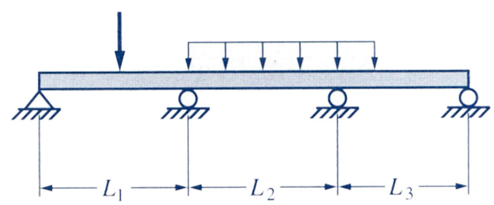
(c) Cantilever beam



(d) Propped cantilever beam



(e) Fixed beam



(f) Continuous beam

Conditions of Equilibrium:

$\sum F_x = 0$

$\sum F_y = 0$

$\sum M_A = 0$ (about any point)

The three equations can be used to solve for no more than three unknowns.

Statically Determinate Beams. In the first three types of beams, shown in a, b, and c, there are three unknown reaction components that may be determined from the static equilibrium equations. Such beams are said to be statically determinate.

Statically Indeterminate Beams. When the number of unknown reaction components exceeds three, as in the beams shown in d, e, and f, the three equilibrium equations are insufficient for determining the unknown reaction components. Such beams are said to be statically indeterminate.

13-3
Types of Loading

Beams are subjected to various loads. Only the concentrated, uniform, and linearly varying loads will be discussed here.

Concentrated Loads

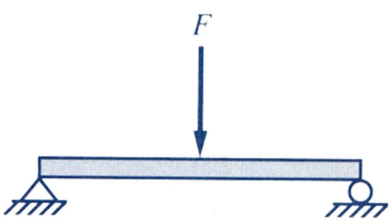
A concentrated load is applied at a specific point on the beam and is considered as a discrete force acting at the point, as shown in (a). For example, a weight fastened to a beam by a cable applies a concentrated load to the beam.

Uniform Loads

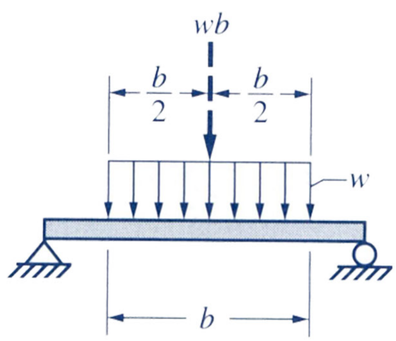
When a load is distributed over a part or the entire length of the beam, it is called a distributed load. If the intensity of a distributed load is a constant value, it is called a uniform load. The load intensity is expressed as force per unit length of the beam, such as lb/ft or N/m. For computing the reactions, the distributed load may be replaced by its equivalent force. The equivalent force of a uniform load is equal to the load intensity w multiplied by the length of distribution b, and the line of action of the equivalent force passes through the midpoint of the length b, as shown in (b). The weight of a beam is an example of a uniformly distributed load.

Linearly Varying Loads

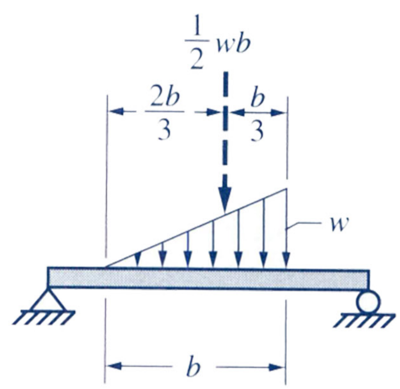
A linearly varying load is a distributed load with a uniform variation of intensity. Such a load condition occurs on a vertical or inclined wall due to liquid pressure. Example (c) shows a linearly varying load, with intensity varying linearly from zero to a maximum value w.



(a) Concentrated load

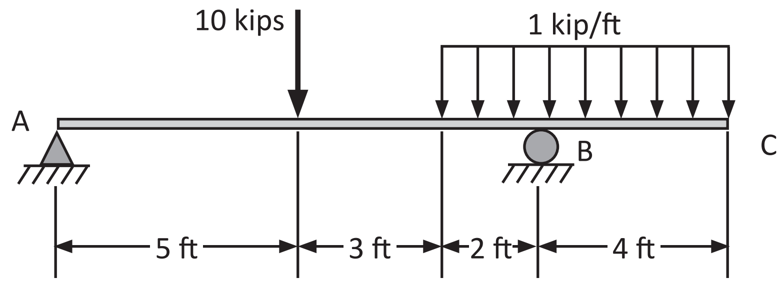


(b) Uniform load



(c) Linearly varying load

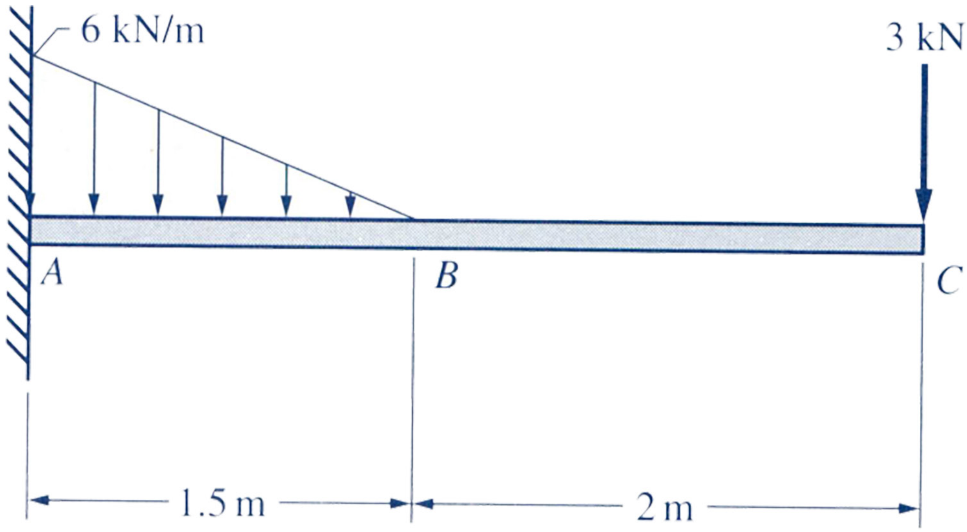
Example 13-1
Determine the external reactions at the supports A and B for the overhanging beam due to the loading shown.



Solution.

Example 13-2

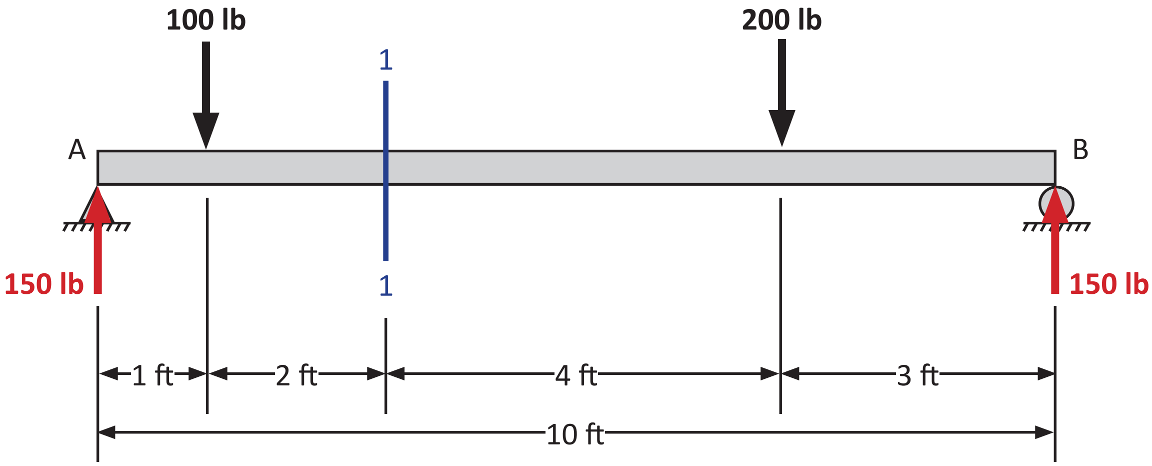
Determine the external reactions at the fixed support of the catilever beam due to the loading shown.

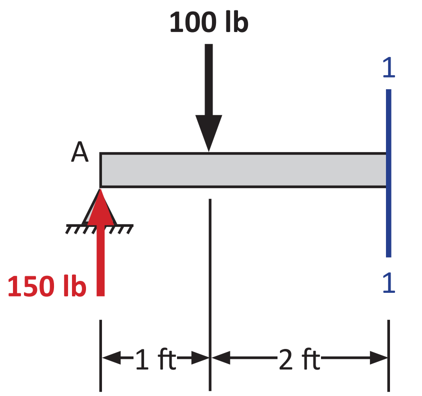


Solution.

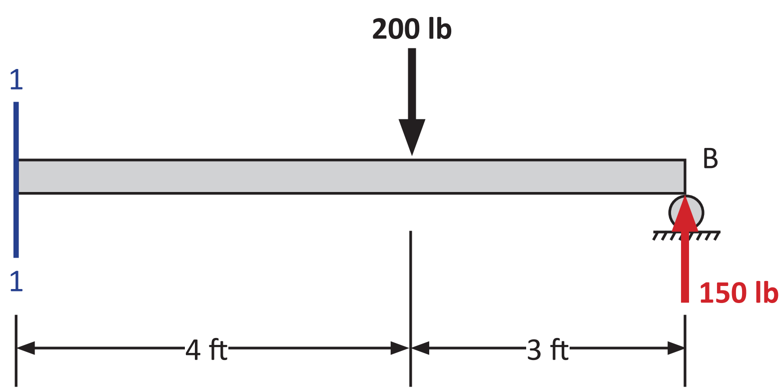
13-5
Shear Force and Bending Moment in Beams

Internal shear force and bending moment are developed in a beam to resist the external forces and to maintain equilibrium.

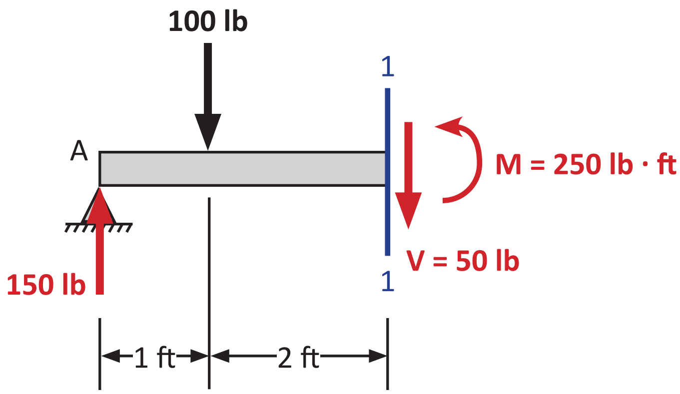


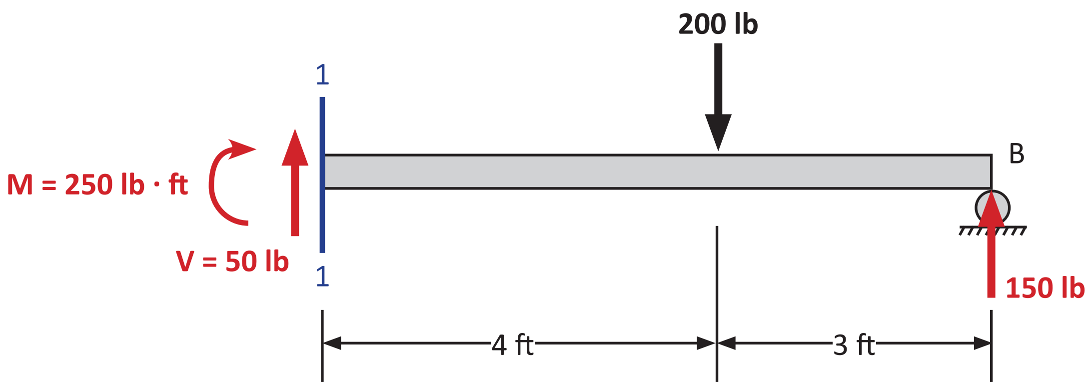


FBD - Left Portion of Section 1-1



FBD - Right Portion of Section 1-1

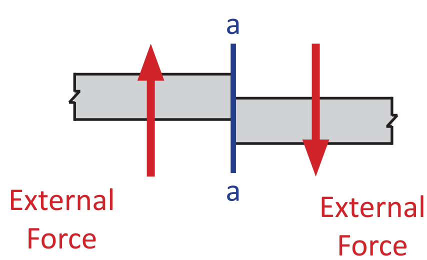




Beam Sign Conventions

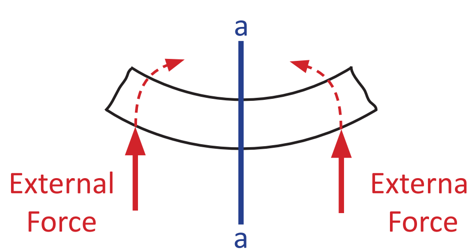
Signs for the internal shear forces and bending moments are based on the effects that they produce:

1. Positive Shear. The shear force at a section is positive if the external forces on the beam produce a shear effect that tends to cause the left side of the section to move up relative to the right side.

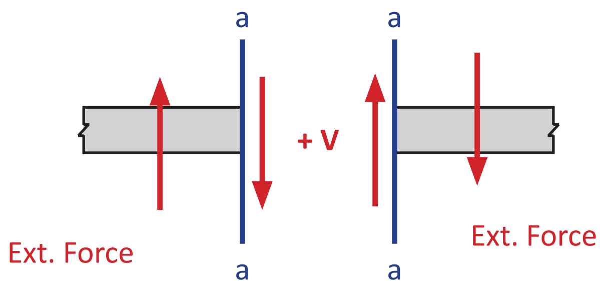


Effect of Positive Shear

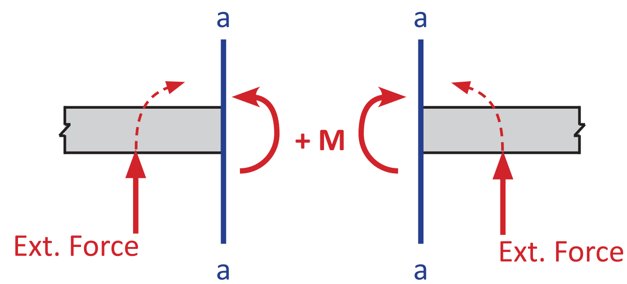
2. Positive Moment. The bending moment of a section is positive if the external forces on the beam produce a bending effect that causes the beam to bend concave upward (the center of curvature is above the curve) at the section.



Effect of Positive Moment



Direction of Positive Internal Shear Force V



Direction of Positive Internal Bending Moment M

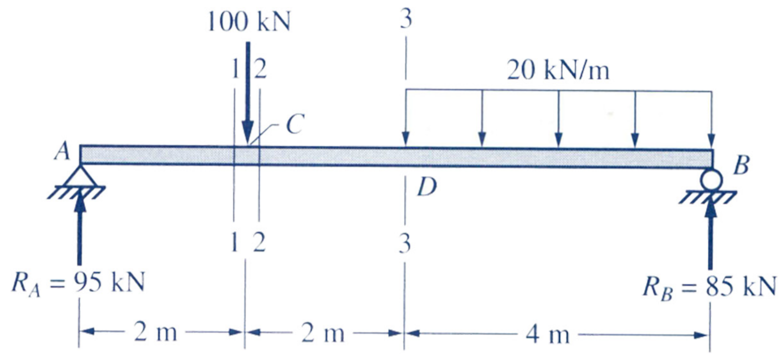
Rule 1 **(For Finding Shear Forces)** The internal shear force at any section of a beam is equal to the algebraic sum of the external forces on either segment separated by the section. If the summation is from the left end of the beam to the section, treat the upward forces as positive. If the summation is from the right end of the beam to the section, treat the downward forces as positive.

V = Σ Ext. Forces {
- From Left: Upward force as positive
- From Right: Downward force as positive

Rule 2 **(For finding Bending Moment)** The internal bending moment at any section of a beam is equal to the algebraic sum of the moments about the section due to the external forces on either segment separated by the section, In either case, treat the moment produced by upward forces as positive.

M = Σ Moments of Ext. Forces {
- From either side: Moment due to upward force as positive

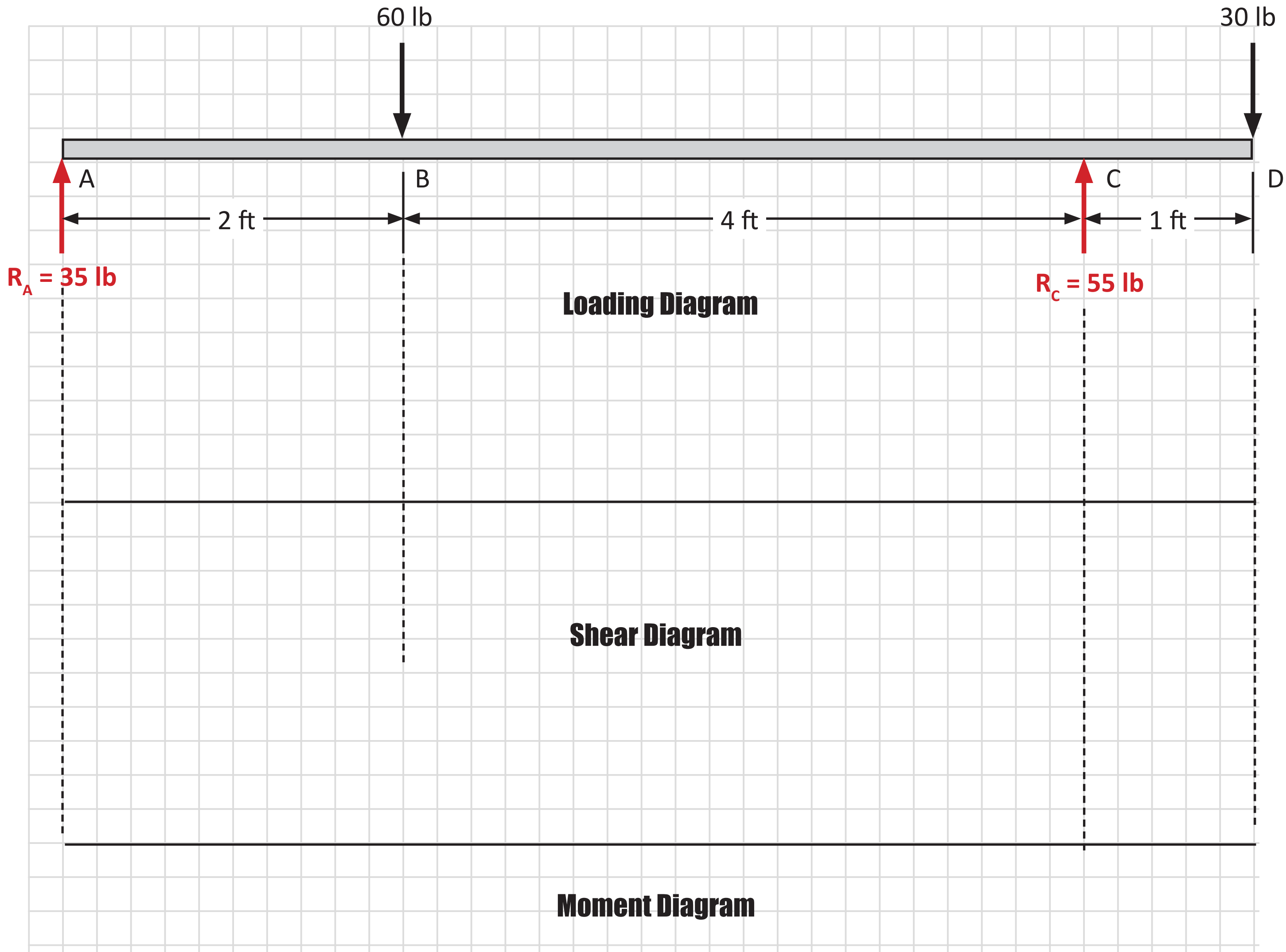
Example 13-3 Calculate the shear forces and bending moments at sections C and D of the beam shown.



Solution.

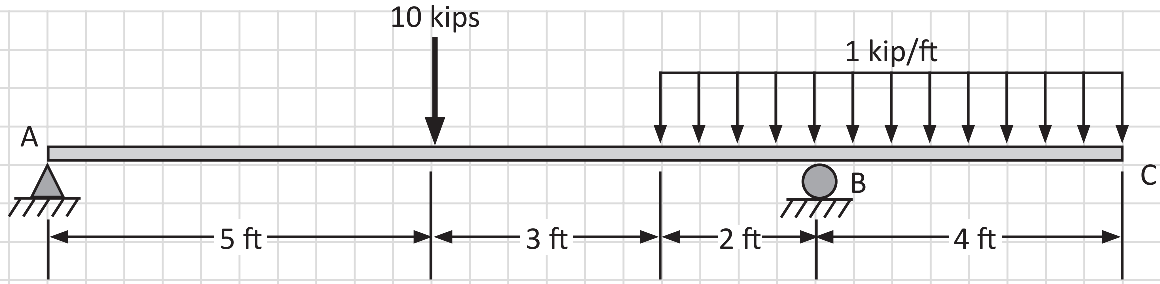
- Shear force and bending moment diagrams depict the variation of shear force and bending moment along the beam.
- Beam sign conventions must be used for plotting the shear force and bending moment diagrams.
- Positive Shear or moment are plotted above the baseline; negative shear or moment are plotted below the baseline.

Example



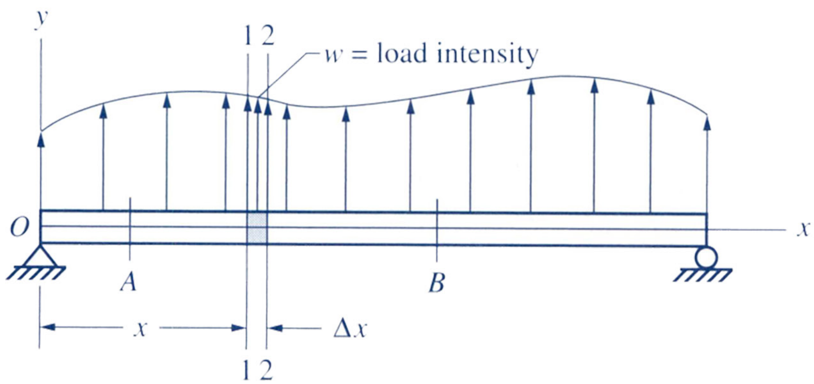
Example

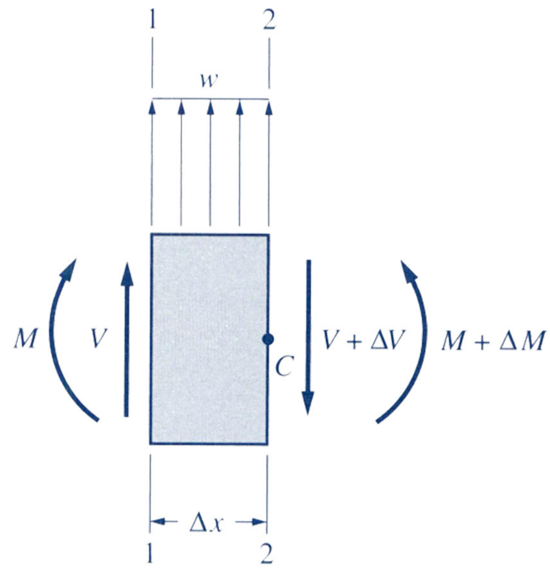
Draw the shear force and bending moment diagrams for Ex 13-1.



Solution.

13-7
Relationships Among Load, Shear, and Moment





Loading Diagram

FBD - Incremental Section (Δx)

Relationship Between Load and Shear

Relationship between Shear and Moment

13-8
Sketching Shear and Moment Diagrams Using Their Relationships

The relationships established in the previous section may be used to facilitate the sketching of shear force and bending moment diagrams.

Loading Diagram. Show all the applied forces and reactions on the beam, including all the relevant dimensions along the beam. Never replace a distributed load by its equivalent concentrated force.

Shear Diagram. The following procedure may be followed for sketching the shear diagram:

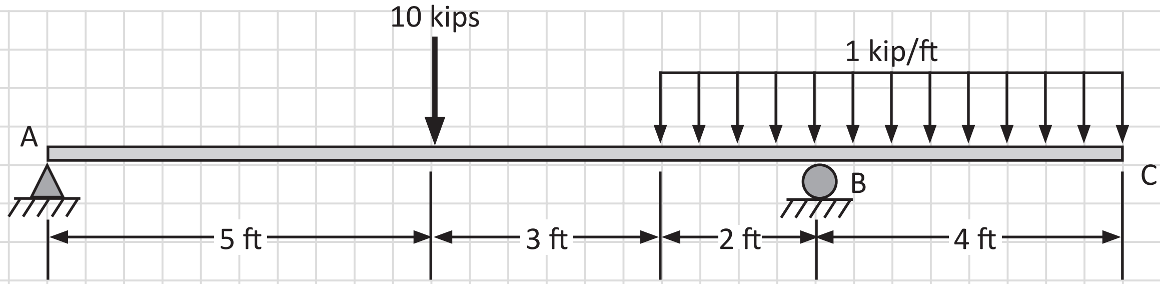
1. For convenience and clarity, the shear diagram should be drawn directly below the loading diagram. A horizontal baseline for the shear diagram is drawn at a proper location below the loading diagram. Draw lines vertically downward from controlling sections, including the sections at the supports, sections at the concentrated forces, and the beginning and end of a distributed load.
2. Starting at the left end, compute the shear at the controlling sections using Equation 13-5. Note that at the section where a concentrated force is applied, the shear force diagram has an abrupt change of values equal to the concentrated load. An upward concentrated load causes an abrupt increase; a downward load causes an abrupt decrease.
3. Plot points on the shear diagram using the shear force of each controlling section as the ordinate. A positive value is plotted above the baseline; a negative value is plotted below the baseline.
4. Connect the adjacent points plotted, and keep in mind that the slope of the shear diagram is equal to the load intensity. The shear diagram is horizontal for the segment of the beam that is not loaded. At the segment of the beam where a downward uniform load is applied, the shear diagram is an inclined line with a downward slope. If the inclined line intersects the baseline, the shear force at the point is zero. Find the location of this point.

Moment Diagram. The following procedure may be followed for sketching the moment diagram:

1. The moment diagram is usually drawn directly under the shear diagram using the same horizontal scale. A horizontal baseline for the moment diagram is drawn at a proper location below the shear diagram. The controlling sections for the moment diagram include those used in sketching the shear diagram plus the section where the shear is zero or where the shear changes sign.
2. Calculate all the areas under the shear diagram between the adjacent controlling sections.
3. Note that the moments at the free end or the ends of a simple beam are always equal to zero. Starting at the left end, compute the moment at the controlling sections using Equation 13-7.

Example

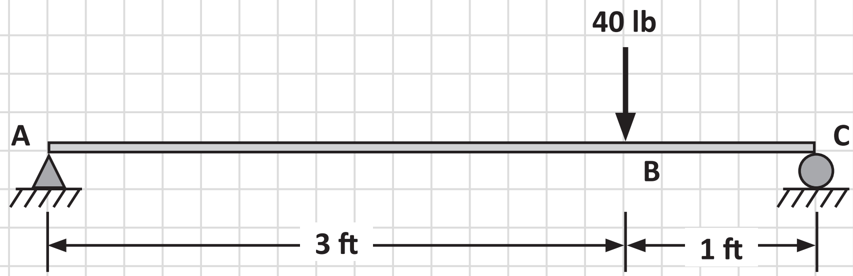
Draw the shear force and bending moment diagrams for Ex 13-1.



Solution.

Example

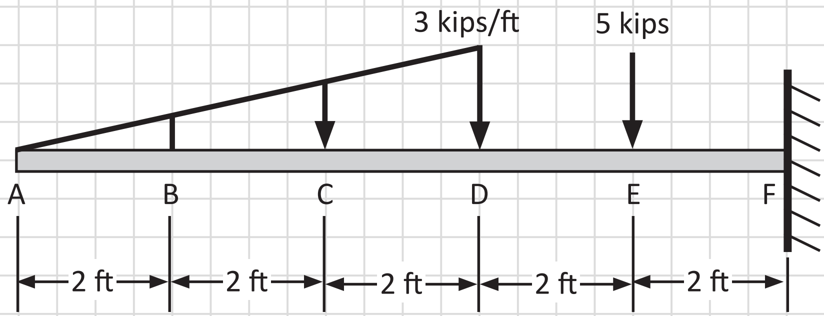
Draw the shear force and bending moment diagrams for the beam subjected to the loading shown.



Solution.

Example

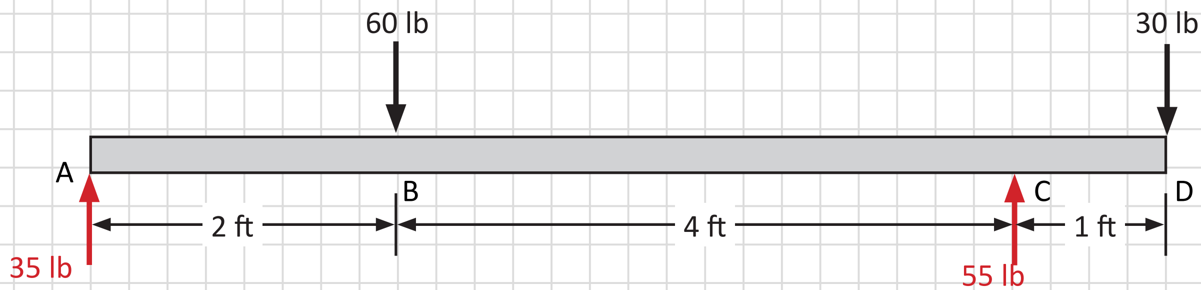
Draw the shear force and bending moment diagrams for the beam subjected to the loading shown.



Solution.

Example

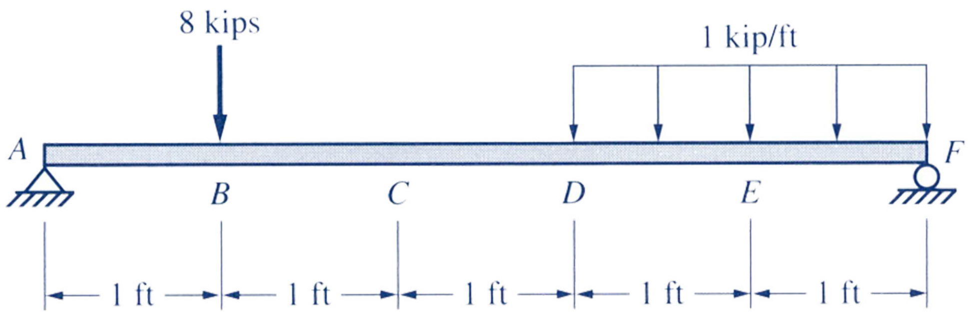
Draw the shear force and bending moment diagrams for the beam subjected to the loading shown.



Solution.

Example

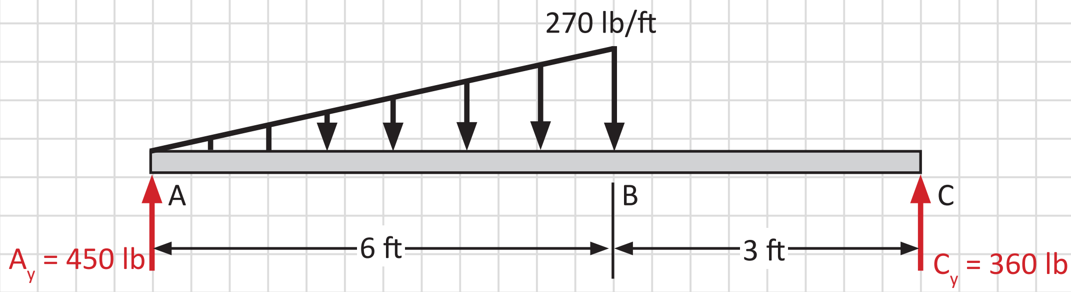
Determine the shear forces and bending moments at section A, B, C, D, E, and F for the beam subjected to the loading shown.



Solution.

Example

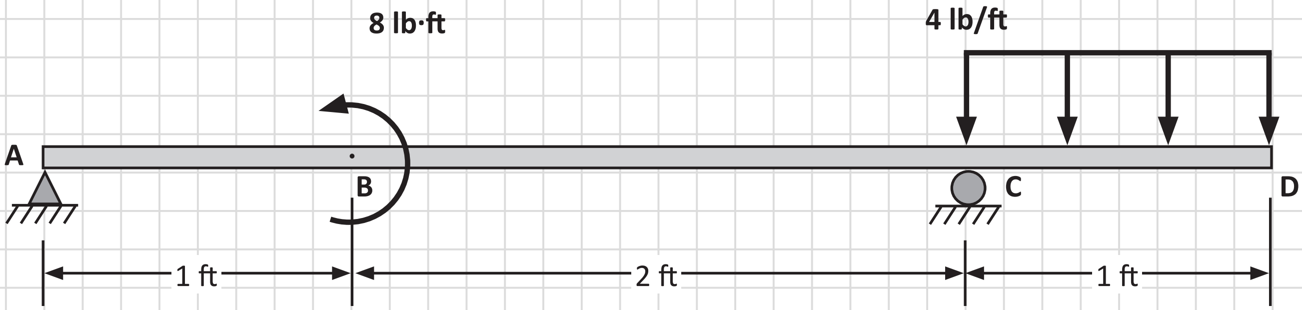
Draw the shear force and bending moment diagrams for the beam subjected to the loading shown.



Solution.

Example

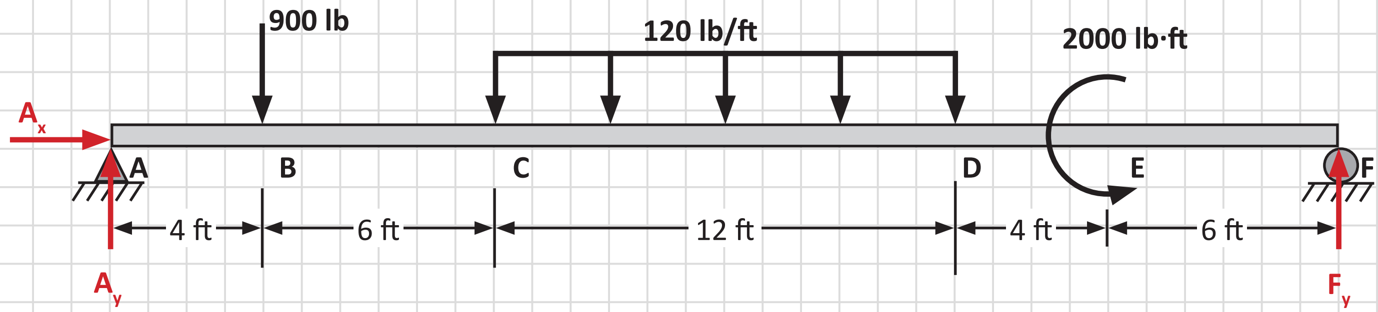
Draw the shear force and bending moment diagrams for the beam subjected to the loading shown.



Solution.

Example

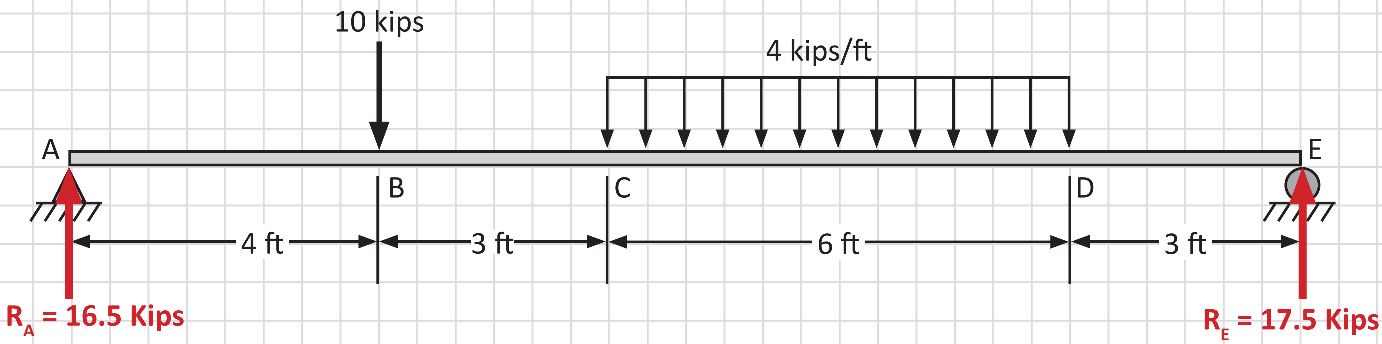
Draw the shear force and bending moment diagrams for the beam subjected to the loading shown. Find the maximum bending moment.



Solution.

Example

Draw the shear force and bending moment diagrams for the beam subjected to the loading shown. Find the maximum shear force and the maximum bending moment.



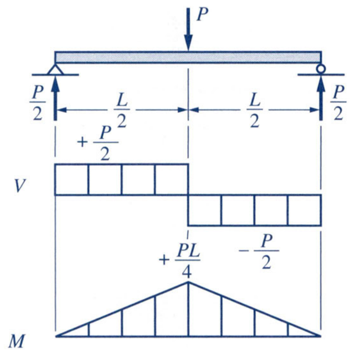
Solution.

13-9

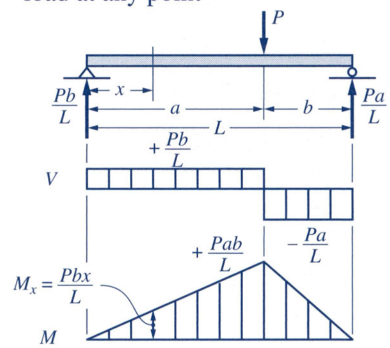
Shear and Moment Formulas

TABLE 13–1 Shear and Moment Formulas for Some Simple Loadings

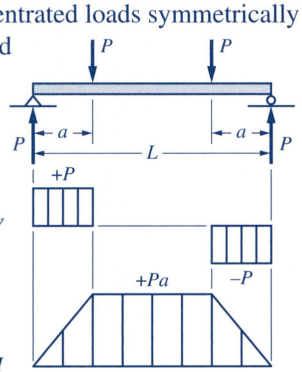
1. Simple beam with a concentrated load at the center



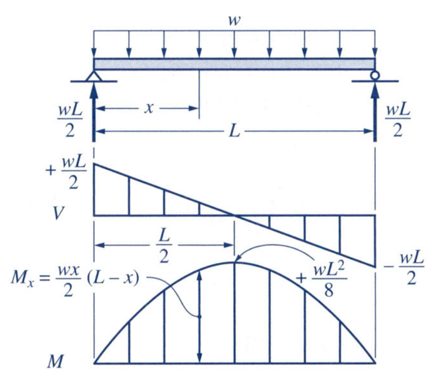
2. Simple beam with a concentrated load at any point



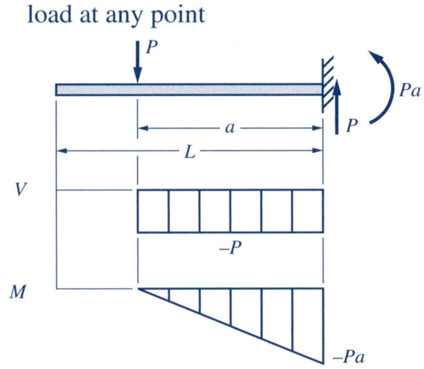
3. Simple beam with two equal concentrated loads symmetrically placed



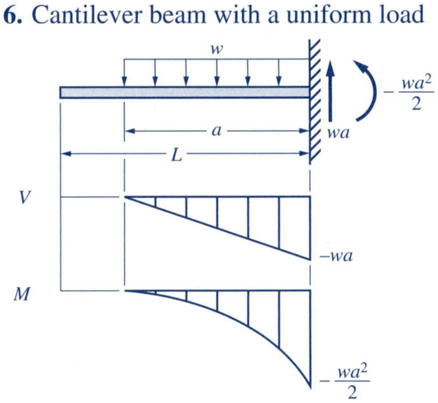
4. Simple beam with a uniform load



5. Cantilever beam with a concentrated load at any point

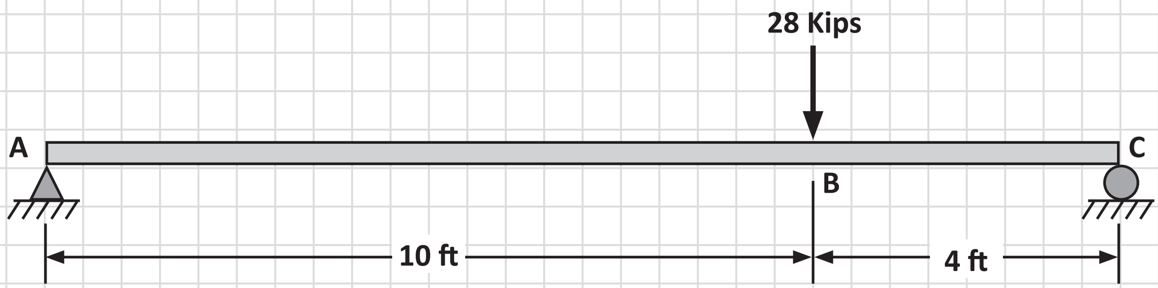


6. Cantilever beam with a uniform load



Example

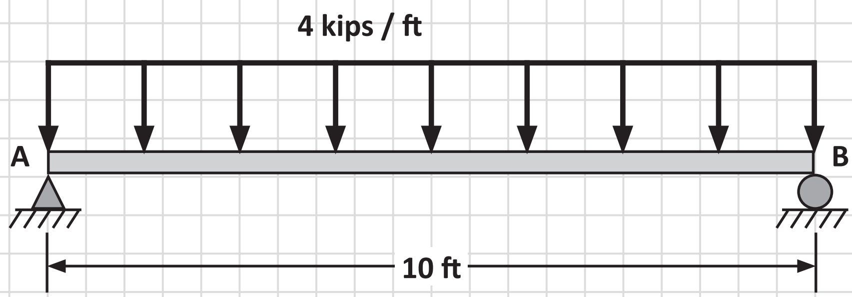
Draw the shear force and bending moment diagrams for the beam subjected to the loading shown. Find the maximum shear force and the maximum bending moment.



Solution.

Example

Draw the shear force and bending moment diagrams for the beam subjected to the loading shown. Find the maximum shear force and the maximum bending moment.

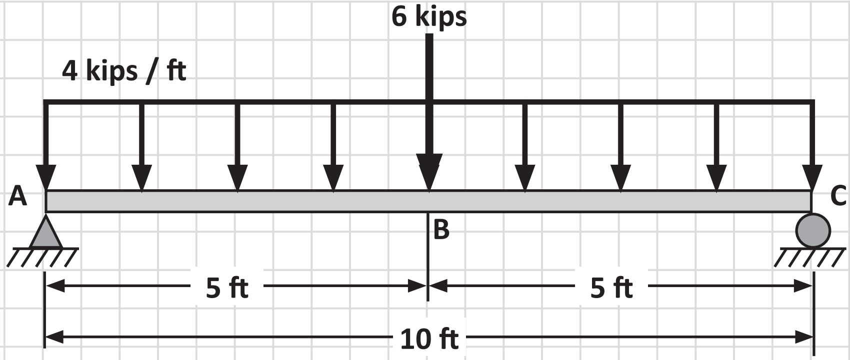


Solution.

The Method of Superposition

- If the maximum shear or the maximum moment is required for a beam subjected to a loading consisting of several forces, the method of superposition can be used.
- Using this method, the effect of each load is computed separately and the combined effect is added algebraically.

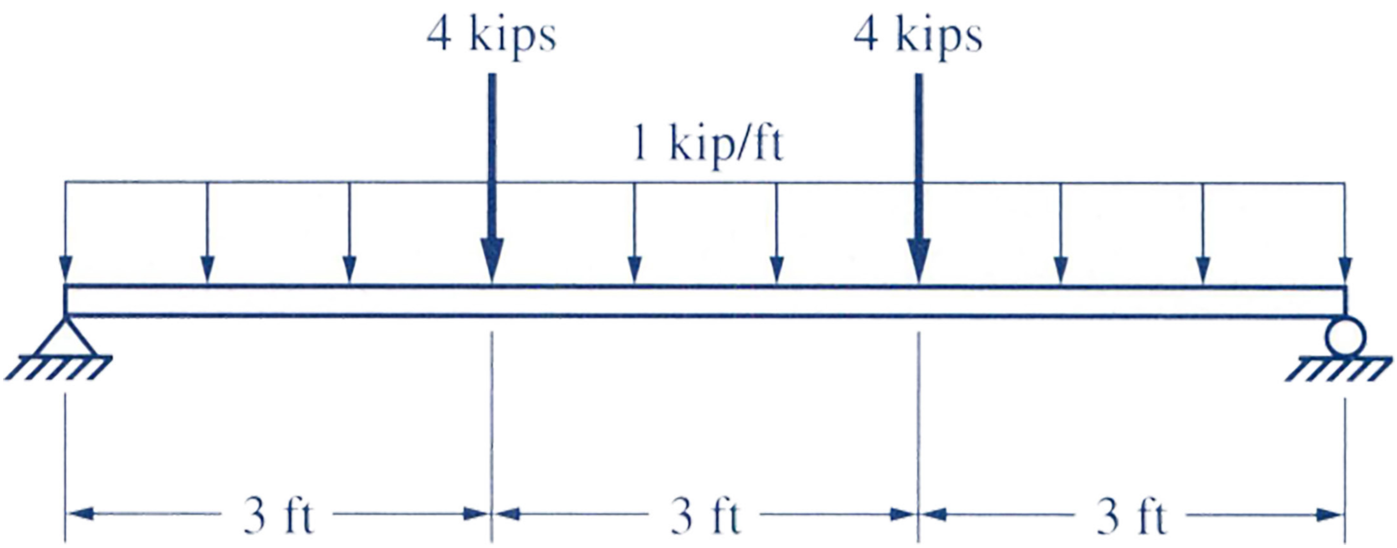
Example



Solution.

Example 13-10

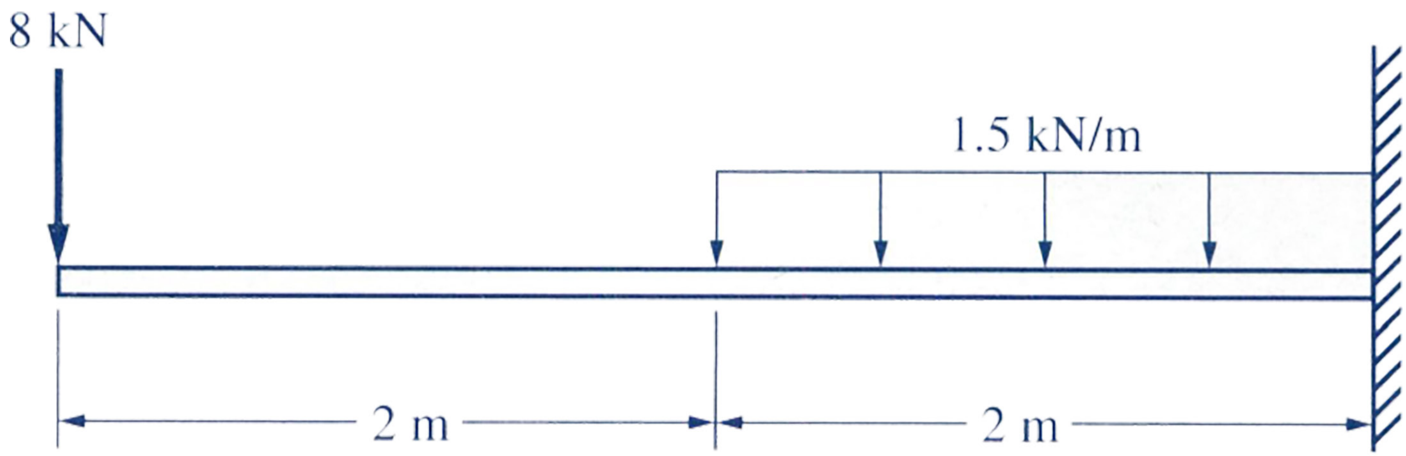
Find the maximum shear force and the maximum bending moment in the simple beam due to the loading shown.



Solution.

Example 13-11

Find the maximum shear force and the maximum bending moment in the simple beam due to the loading shown.



Solution.