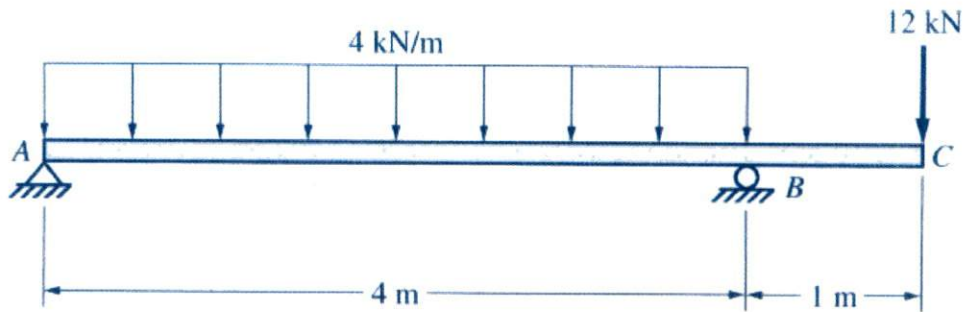


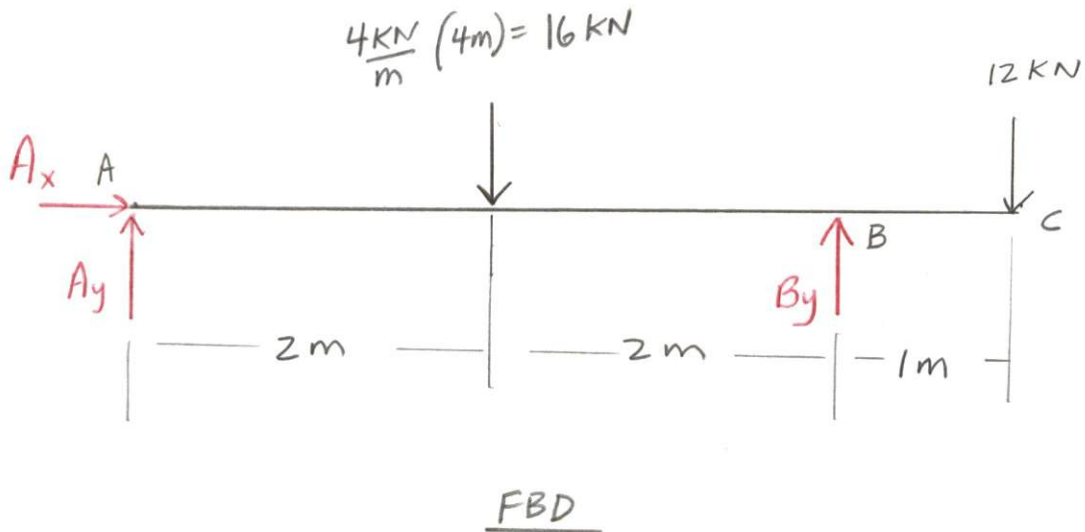
13-17

13-16 to 13-21 Refer to Figs. P13-16 to P13-21. Draw the shear force and bending moment diagrams for each beam. Locate the section with zero shear force (if any) and determine the moment at the section.



Solution.

Step 1 solve for the reactions at the supports A and B.



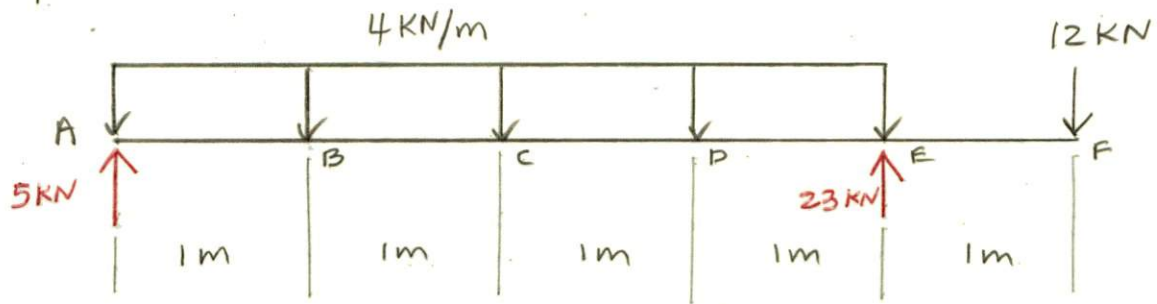
Equilibrium Equations

ccw + M ↺
cw - M ↻

$$[\sum F_x = 0] \quad A_x = 0$$

$$[\sum M_A = 0] \quad -16 \text{ kN}(2\text{m}) + B_y(4\text{m}) - 12 \text{ kN}(5\text{m}) = 0$$
$$B_y = \frac{92 \text{ kN}\cdot\text{m}}{4\text{m}} = 23 \text{ kN} \uparrow$$

$$[\sum F_y = 0] \quad A_y - 16 \text{ kN} + B_y - 12 \text{ kN} = 0$$
$$A_y = 28 \text{ kN} - 23 \text{ kN} = 5 \text{ kN} \uparrow$$

Loading DiagramShear Force (V)

$$V_{A-} = 0$$

$$V_{A+} = 5 \text{ kN}$$

$$V_B = 5 \text{ kN} - 4 \frac{\text{kN}}{\text{m}} (1 \text{ m}) = 5 \text{ kN} - 4 \text{ kN} = 1 \text{ kN}$$

$$V_C = 5 \text{ kN} - 4 \frac{\text{kN}}{\text{m}} (2 \text{ m}) = 5 \text{ kN} - 8 \text{ kN} = -3 \text{ kN}$$

$$V_D = 5 \text{ kN} - 4 \frac{\text{kN}}{\text{m}} (3 \text{ m}) = 5 \text{ kN} - 12 \text{ kN} = -7 \text{ kN}$$

$$V_{E-} = 5 \text{ kN} - 4 \frac{\text{kN}}{\text{m}} (4 \text{ m}) = 5 \text{ kN} - 16 \text{ kN} = -11 \text{ kN}$$

$$V_{E+} = 5 \text{ kN} - 4 \frac{\text{kN}}{\text{m}} (4 \text{ m}) + 23 \text{ kN} = -11 \text{ kN} + 23 \text{ kN} = +12 \text{ kN}$$

$$V_{F-} = +12 \text{ kN}$$

$$V_{F+} = +12 \text{ kN} - 12 \text{ kN} = 0$$

Bending Moment (M)

$$M_A = 0$$

$$M_B = 5 \text{ kN}(1 \text{ m}) - 4 \frac{\text{kN}}{\text{m}} (1 \text{ m}) (0.5 \text{ m}) = +3 \text{ kN}\cdot\text{m}$$

$$M_C = 5 \text{ kN}(2 \text{ m}) - 4 \frac{\text{kN}}{\text{m}} (2 \text{ m}) (1 \text{ m}) = +2 \text{ kN}\cdot\text{m}$$

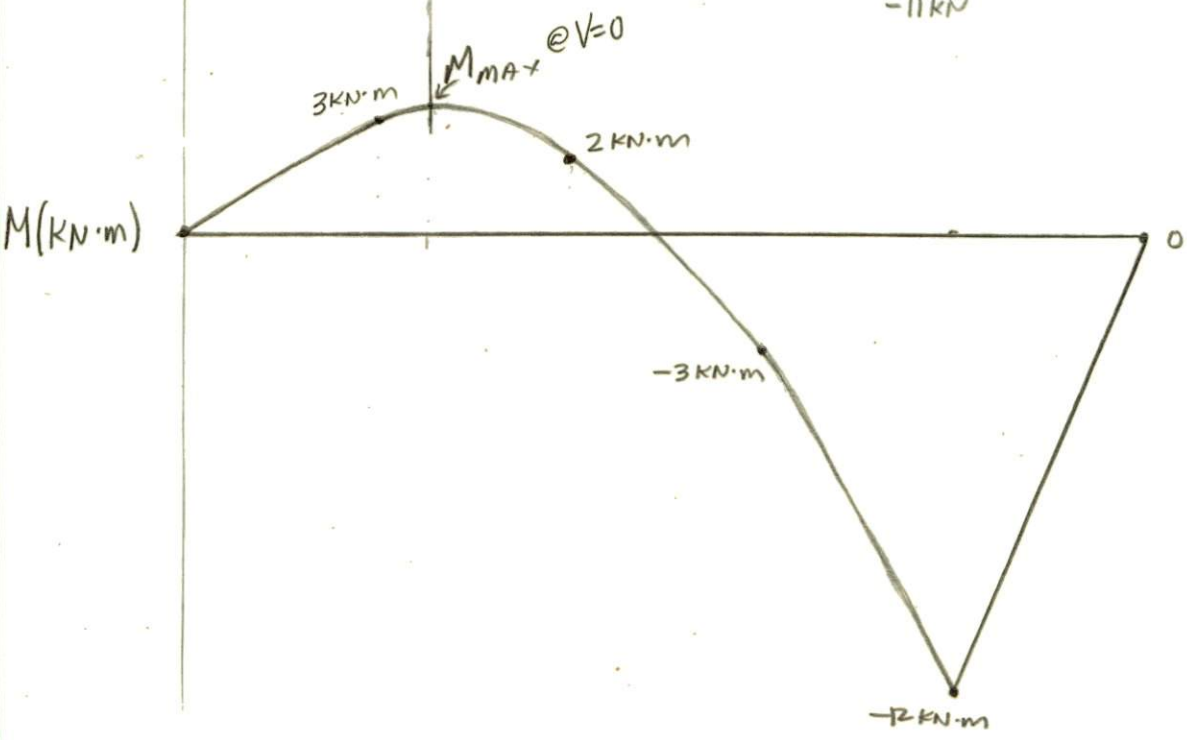
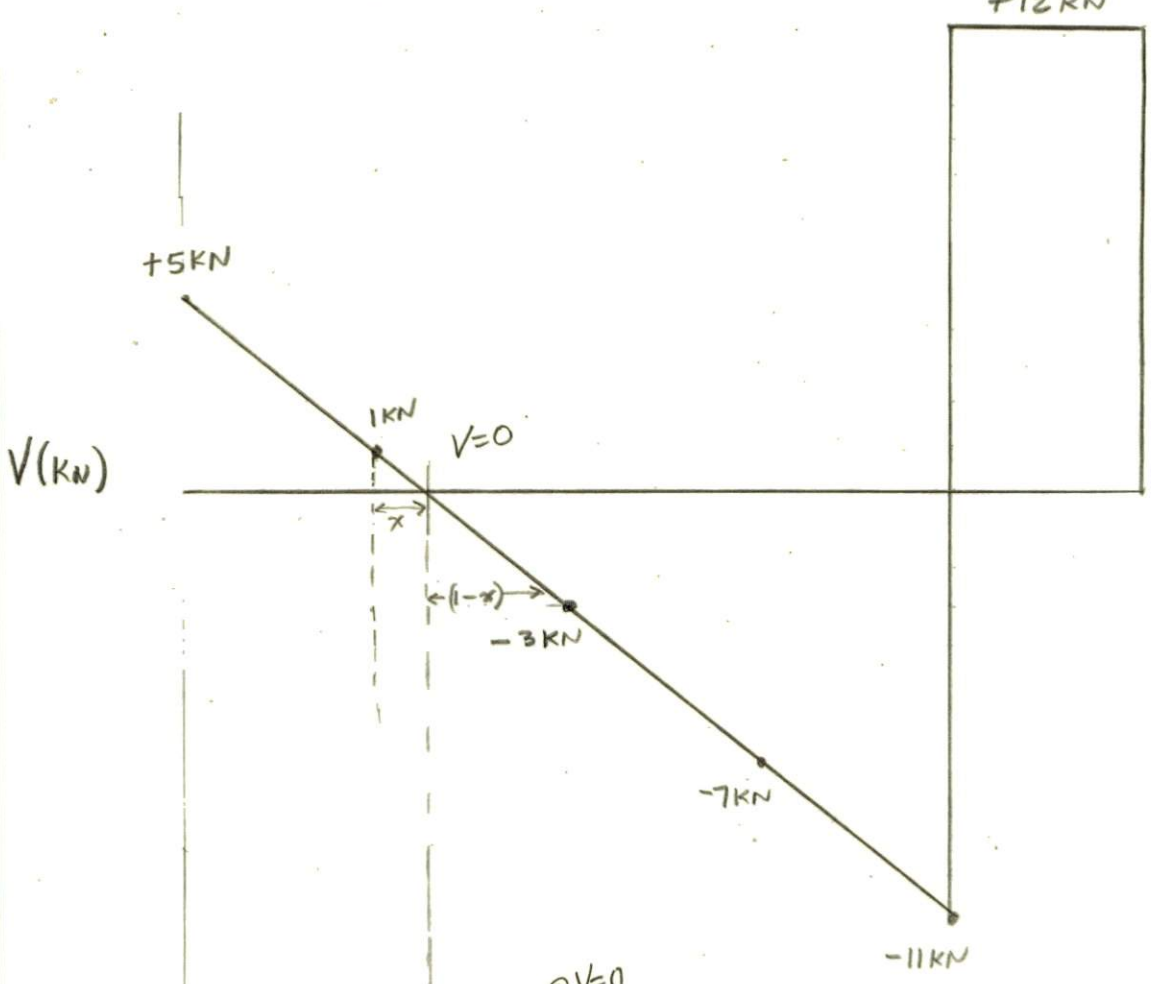
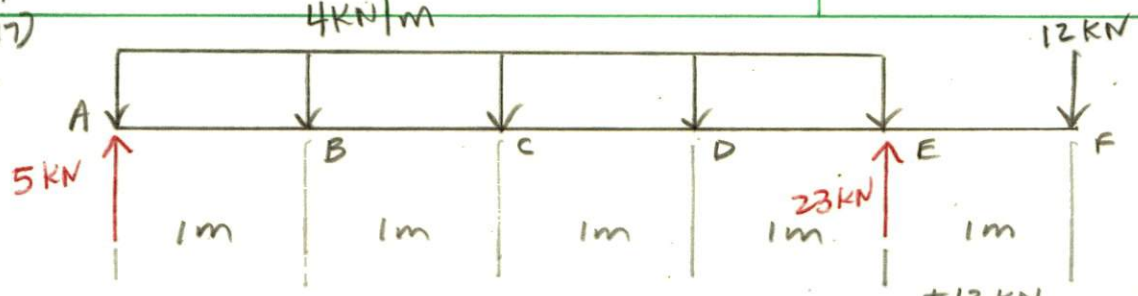
$$M_D = 5 \text{ kN}(3 \text{ m}) - 4 \frac{\text{kN}}{\text{m}} (3 \text{ m}) (1.5 \text{ m}) = -3 \text{ kN}\cdot\text{m}$$

$$M_E = 5 \text{ kN}(4 \text{ m}) - 4 \frac{\text{kN}}{\text{m}} (4 \text{ m}) (2 \text{ m}) = -12 \text{ kN}\cdot\text{m}$$

$$M_F = 5 \text{ kN}(5 \text{ m}) - 4 \frac{\text{kN}}{\text{m}} (4 \text{ m}) (3 \text{ m}) + 23 \text{ kN}(1 \text{ m}) = 0$$

Step 3 Shear / Moment Diagrams

(13-17)



The maximum bending moment occurs when $V=0$

Find M_{max}

Method 1

Equation of a straight line

$$y = mx + b$$

$$b = 5 \text{ kN}$$

$$m = -\frac{16 \text{ kN}}{4 \text{ m}} = -4 \frac{\text{kN}}{\text{m}}$$

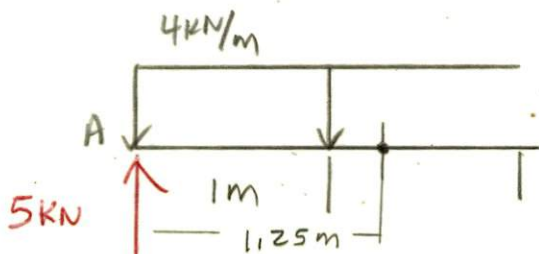
$$y = -4 \frac{\text{kN}}{\text{m}} x + 5 \text{ kN}$$

$$y = 0 \quad 0 = -4 \frac{\text{kN}}{\text{m}} x + 5 \text{ kN}$$

$$-5 \text{ kN} = -4 \frac{\text{kN}}{\text{m}} x$$

$$x = \frac{5 \text{ kN} \cdot \text{m}}{4 \text{ kN}} = 1.25 \text{ m}$$

Determine M @ $x = 1.25 \text{ m}$



$$\begin{aligned} M_{max} &= 5 \text{ kN}(1.25 \text{ m}) - \frac{4 \text{ kN}}{\text{m}} (1.25 \text{ m}) \left(\frac{1.25 \text{ m}}{2} \right) \\ &= 6.25 \text{ kN} \cdot \text{m} - 3.125 \text{ kN} \cdot \text{m} \\ &= \underline{\underline{3.125 \text{ kN} \cdot \text{m}}} \end{aligned}$$

Method 2

By similar triangles (shear Diagram)

$$\frac{x}{1-x} = \frac{1}{3}$$

$$3x = 1-x$$

$$4x = 1$$

$$x = \frac{1}{4} = 0.25 \text{ m}$$

M_{\max} occurs @ $x = 1.25 \text{ m}$

$$\begin{aligned} M_{\max} &= 5 \text{ kN}(1.25 \text{ m}) - \frac{4 \text{ kN}}{\text{m}}(1.25 \text{ m}) \left(\frac{1.25 \text{ m}}{2} \right) \\ &= \underline{\underline{3.125 \text{ kN}\cdot\text{m}}} \end{aligned}$$