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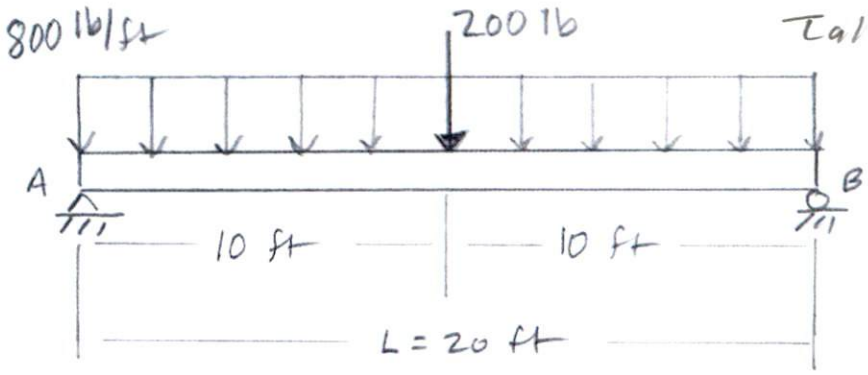
Name: Solution

1. For the rectangular southern pine timber beam in E3P2, (a) find the lightest beam for the load given that will satisfy the allowable deflection of L/240. (b) for the beam selected determine the maximum deflection.

E = 1800 ksi

$\sigma_{allow} = 1600$ psi

$\tau_{allow} = 90$ psi



cross-sectional Area

Solution.

Step 1. List the knowns

$L = 20$ ft

$P = 200$ lb

$w = 800$ lb/ft

$a = 10$ ft

Step 2.

Table 13-1, Case 1 and Case 4

$$V_{MAX} = \frac{P}{2} + \frac{wL}{2} = \frac{200\text{ lb}}{2} + \frac{800\text{ lb/ft} \times 20\text{ ft}}{2} = 8100\text{ lb}$$

$$M_{MAX} = \frac{PL}{4} + \frac{wL^2}{8} = \frac{200\text{ lb}(20\text{ ft})}{4} + \frac{800\frac{\text{lb}}{\text{ft}}(20\text{ ft})^2}{8} = 41{,}000\text{ lb-ft}$$

Step 3.

$$S_{req} = \frac{M_{MAX}}{\sigma_{allow}} = \frac{41{,}000\text{ lb ft} \times \frac{12\text{ in}}{1\text{ ft}}}{1600\text{ psi}} = 307.5\text{ in}^3$$

Step 4.

$$A_{req} = \frac{1.5\,V_{MAX}}{\tau_{allow}} = \frac{1.5\,(8100\text{ lb})}{90\text{ psi}} = 135\text{ in}^2$$

Step 5.

8×20	$A = 146\ in^2$	$S = 475\ in^3$	40.6
10×16	$A = 147\ in^2$	$S = 380\ in^3$	40.9

Try 8×20 (Lightest)

$$M_{WT} = \frac{wL^2}{8} = \frac{40.6\ lb/ft\ (20\ ft)^2}{8} = 2030\ lb\cdot ft$$

$$\frac{M_{WT}}{M_{MAX}} = \frac{2030\ lb\cdot ft}{41{,}000\ lb\ ft} = 0.0495 = 4.95\%$$

$$\frac{Extra\ S}{S_{req}} = \frac{475\ in^3 - 307.5\ in^3}{307.5\ in^3} = 0.5447 = 54.47\% > 4.95\%$$

✓ Bending

$$\frac{Extra\ A}{A_{req}} = \frac{146\ in^2 - 135\ in^2}{135\ in^2} = 0.0815 = 8.15\% \quad 74.95\%$$

✓ Shear

Check Deflection,

$E = 1800\ ksi$

8×20 $I = 4634\ in^4$ (Table A-6a)

$$\delta_{MAX} = \frac{PL^3}{48EI} + \frac{5wL^4}{384EI}$$

$$= \frac{0.2\ kip\ (240\ in)^3}{48\ (1800\ ksi)\ (4634\ in^4)} + \frac{5\ (0.8\ k/ft)\left(\frac{ft}{12\ in}\right)(240\ in)^4}{384\ (1800\ ksi)\ 4634\ in^4}$$

$$= 0.0069\ in + 0.34527\ in$$

$$= 0.352\ in$$

$$\delta_{MAX} = 0.352\ in < \delta_{allow} = \frac{240\ in}{240} = 1\ in$$

✓ ok for deflection

Use, 8×20