CMGT 350 **Final Exam** **Fall 2021**

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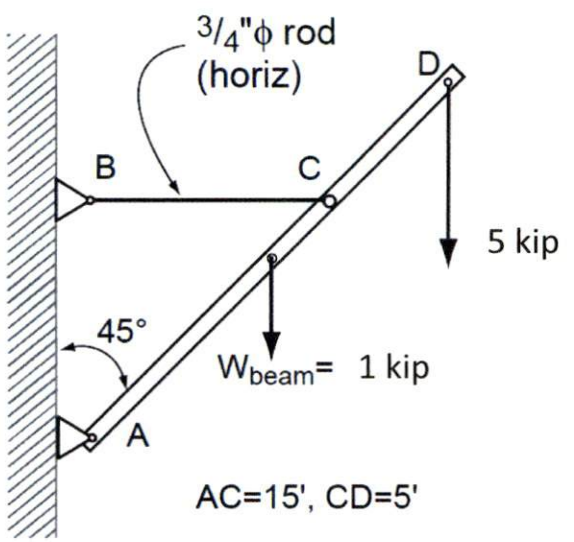
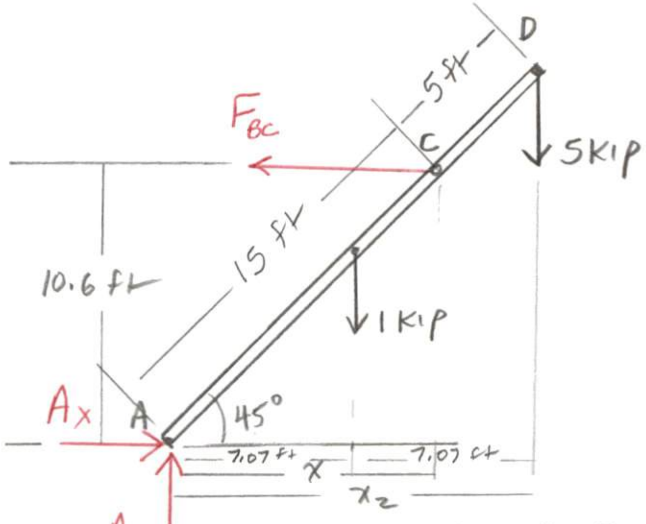
Name: Solution

1. A rigid steel beam 20 ft long supports a 5-kip load as shown. Assume the beam weighs 1 kip, concentrated at the mid-length of the beam.
   A) Determine the tensile stress developed in the steel rod that has a diameter of d = ¾".
   B) What is the minimum diameter rod to the nearest sixteenth of an inch that can safely resist the load if the maximum (allowable) tension stress is equal to σ = 22 ksi?



Solution.
Draw the FBD of Member ACD and solve for F_{BC}



FBD- member ACD

Right-Triangle Math

$$\cos 45^\circ = \frac{x}{15\text{ ft}}$$

$$x = 15\text{ft} \cos 45^\circ = 10.6\text{ ft}$$

$$x_2 = 20\text{ft} \cos 45^\circ = 14.14\text{ ft}$$

Equilibrium Equations

$$+\circlearrowleft[\Sigma M_A = 0] \quad -1\text{ kip}(7.07\text{ ft}) - 5\text{ kip}(14.14\text{ ft}) + F_{BC}(10.6\text{ ft}) = 0$$

$$F_{BC} = \frac{77.77\text{ kip}\cdot\text{ft}}{10.6\text{ ft}} = 7.3\text{ kip}$$

A. $$\sigma = \frac{P}{A} = \frac{7.3\text{ kip}}{\frac{\pi(0.75\text{in})^2}{4}} = \frac{7.3\text{ kip}}{0.442\text{ in}^2} = \underline{\underline{16.5\text{ psi}}}$$

B. $$A = \frac{\pi d^2}{4}$$

$$\sigma = \frac{P}{A}$$

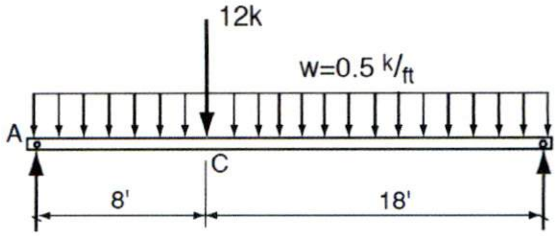
$$A = \frac{P}{\sigma}$$

$$\frac{\pi d^2}{4} = \frac{7.3\text{ kip}}{22\text{ ksi}}$$

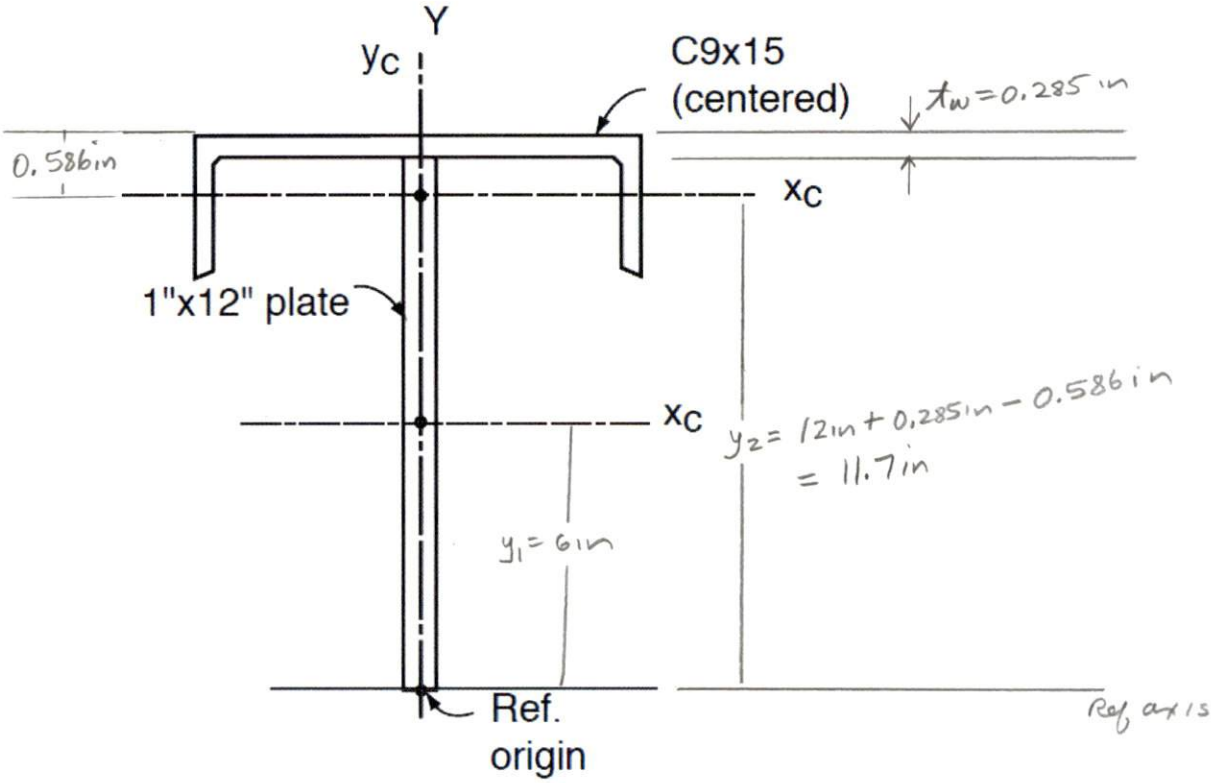
$$d = \sqrt{\frac{4(7.3\text{ kip})}{\pi(22\text{ ksi})}} = 0.65\text{ in}$$

use, $\underline{\underline{11/16\text{ in}}}$ (0.688 in)

2. A built-up beam section (symmetrical about the Y-axis) is used to support a single concentrated load plus a distributed load as shown.



A) Determine the moment of inertia about the centroidal X-X axis.



X-X Axis

Part	A (in^2)	y (in)	Ay (in^3)	$\bar{y}-y$ (in)	$A(\bar{y}-y)^2$ (in^4)	I (in^4)
PL 1X12	12	6	72	1.53	28	144
C 9X15	4.41	11.7	51.597	−4.17	76.7	1.93
	16.41		123.597		105	146

$$\bar{y} = \frac{\Sigma Ay}{\Sigma A} = \frac{123.597\ in^3}{16.41\ in^2} = 7.53\ in$$

$$I_x = \Sigma I + A(\bar{y}-y)^2 = 146\ in^4 + 105\ in^4 = \underline{251\ in^4}$$

B) Using Table 13-1, determine the reactions at the supports A and B.

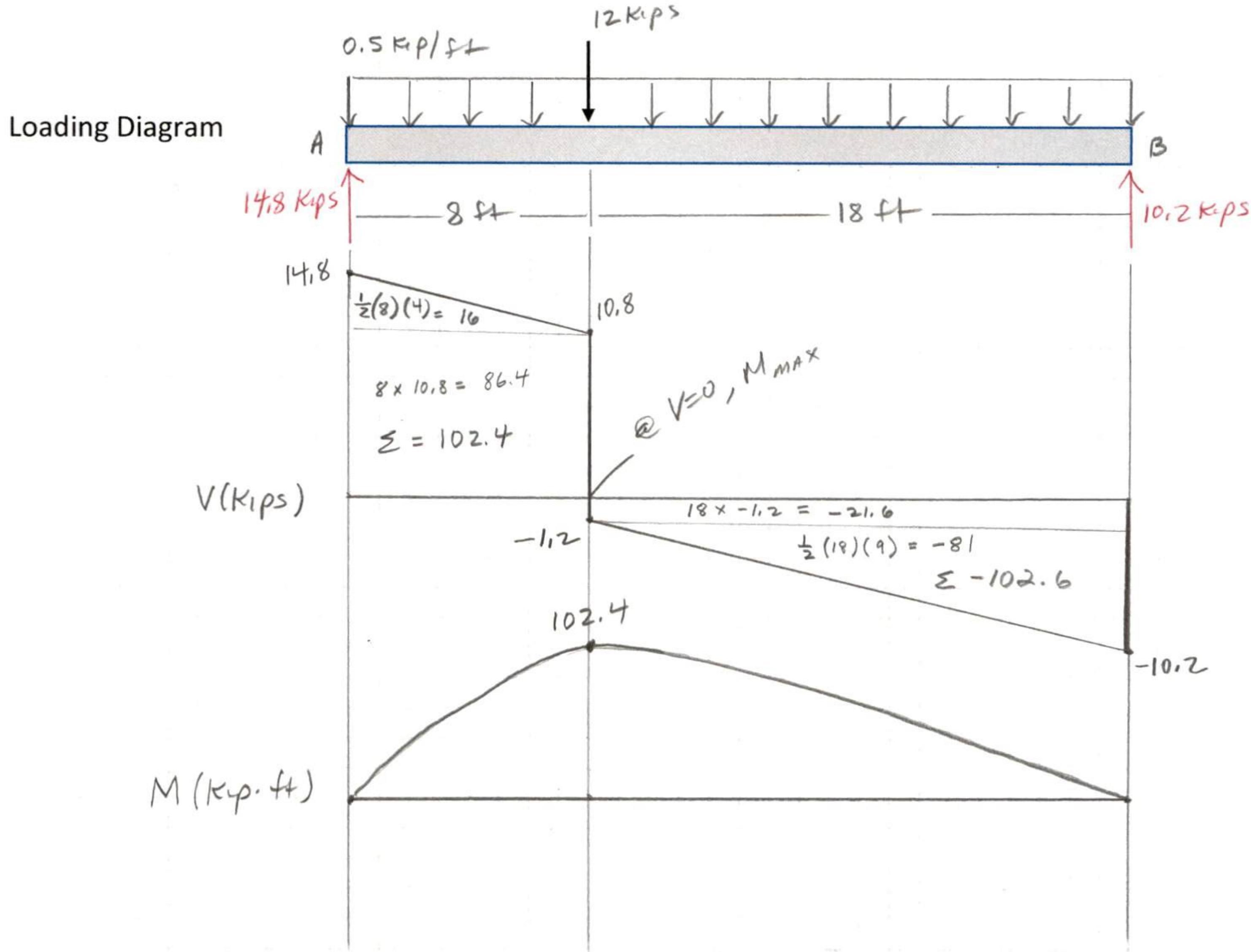
case 2 and case 4 (Superposition)

$$A_y = \frac{Pb}{L} + \frac{wL}{2} = \frac{12\ kip\ (18\ ft)}{26\ ft} + \frac{0.5\ kip/ft\ (26\ ft)}{2} = \underline{14.8\ kip} \uparrow$$

$$B_y = \frac{Pa}{L} + \frac{wL}{2} = \frac{12\ kip\ (8\ ft)}{26\ ft} + 6.5\ kips = \underline{10.2\ kips} \uparrow$$

C) Complete the loading diagram, sketch the V and M diagrams, and locate the section(s) with zero shear and determine the moment(s) at the section(s).

D) Determine the maximum flexural stress developed. Will A36 steel be adequate?

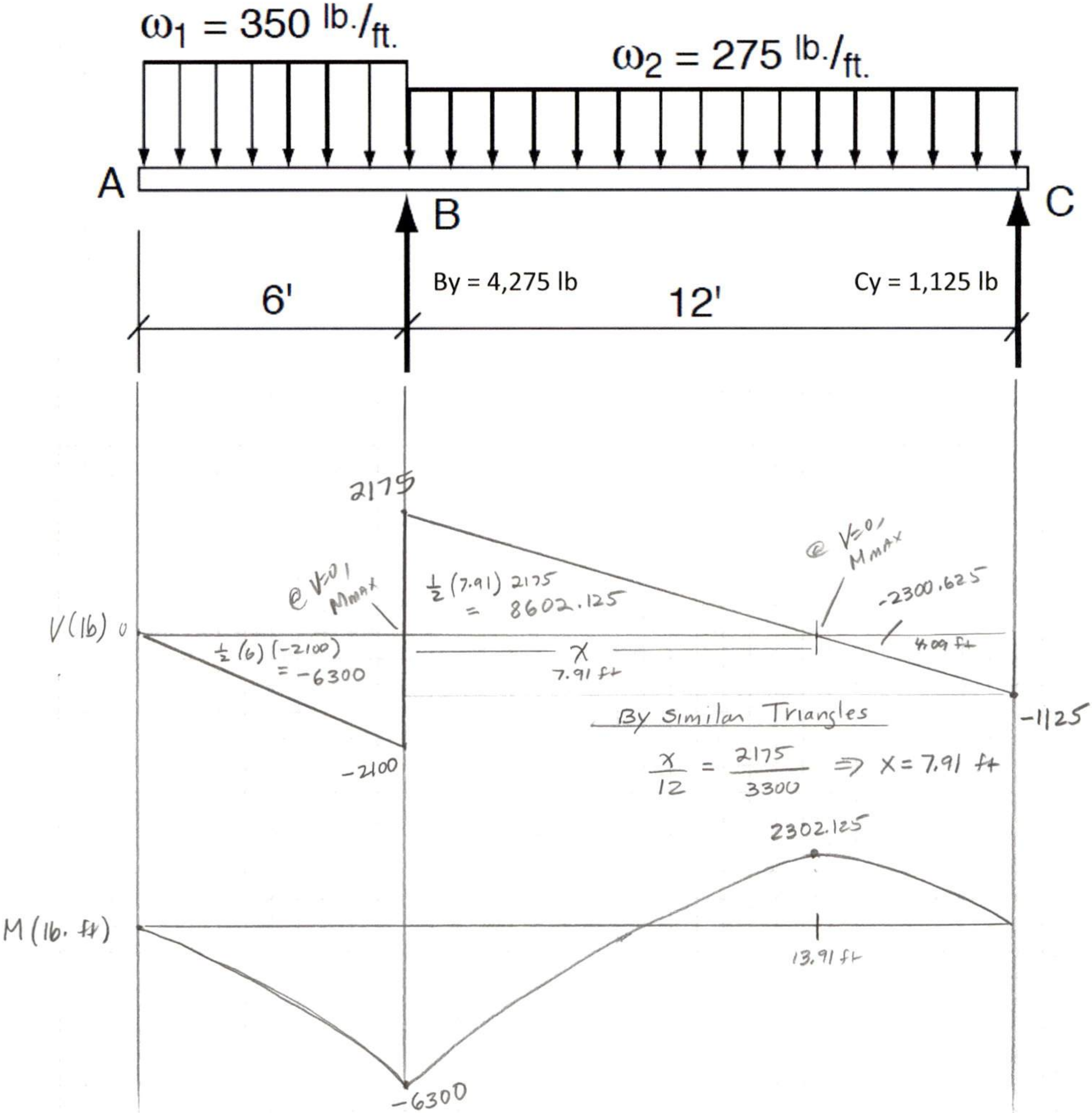


D) $$\sigma = \frac{Mc}{I_x} = \frac{102.4 \text{ kip}\cdot\text{ft}\left(\frac{12\text{ in}}{\text{ft}}\right)(7.53\text{ in})}{251\text{ in}^4}$$

$$= 36.9 \text{ ksi} > \sigma_{allow} = 24 \text{ ksi} \quad (\text{A36 Steel})$$

NOT ADEQUATE

3. Timber roof beams are spaced 5 ft - 0 in O.C. and support a snow load of w_1 = 350 lb/ft and w_2 = 275 lb/ft which includes the total roof dead load (including the framing weight) as shown. A) Draw the shear and bending moment diagrams for the beam due to the loading shown. Locate the section(s) with zero shear and determine the moment(s) at the sections. B) Assuming Hem-Fir No. 1 grade (beams and posts) design the lightest nominal 6 x ____ beam. See table 5-2 Allowable Stresses for Selected Engineering Materials. Assume the beam weight is included in the loads.



$V_{MAX} = 2175 \text{ lb}$

$M_{MAX} = 6300 \text{ lb} \cdot \text{ft}$

HEM-FIR NO. 1 Grade

$\sigma_{allow} = 1000 \text{ psi}$

$\tau_{allow} = 70 \text{ psi}$

$$S_{req} = \frac{M_{MAX}}{\sigma_{allow}} = \frac{6300 \text{ lb} \cdot \text{ft} \left(\frac{12 \text{ in}}{\text{ft}}\right)}{1000 \text{ lb/in}^2} = 75.6 \text{ in}^3$$

$$A_{req} = \frac{1.5 V_{MAX}}{\tau_{allow}} = \frac{1.5(2175 \text{ lb})}{70 \text{ psi}} = 46.6 \text{ in}^2$$

Table A-6(a)

Use, 6 × 10

$S = 82.7 \text{ in}^3$

$A = 52.3 \text{ in}^2$

$wt = 14.5 \text{ lb/ft}$

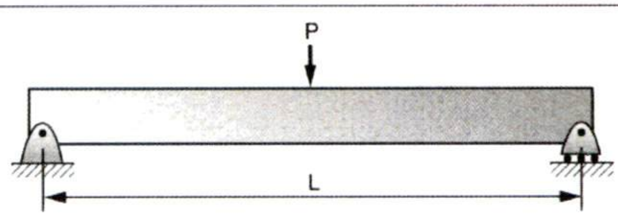
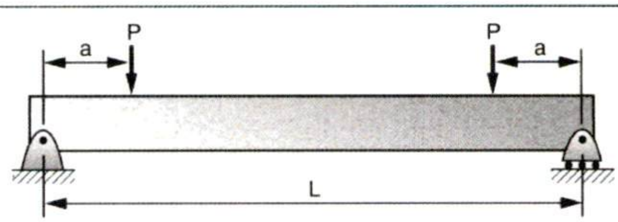
Table 5.2 Allowable stresses for selected engineering materials.

Materials	Unit Weight (density) γ (pfc)	Modules of Elasticity E (ksi)	Allowable Tension Stress F_t	Allowable Axial Compress. F_c	Allowable Compress. Bearing $F_{c\perp}$	Allowable Bending Stress F_b	Allowable Shear Stress F_v
Metals:							
A-36 Steel $F_y = 36$ ksi	490	29,000	22 ksi	22 ksi		22 ksi	14.5 ksi
A-572 Grade 50/A992	490	30,000	30 ksi	30 ksi		30 ksi	20 ksi
A-572 Steel $F_y = 65$ ksi	490	30,000	39 ksi	39 ksi		39 ksi	26 ksi
Aluminum	165	10,000	16 ksi	16 ksi		16 ksi	10 ksi
Iron (cast)	450	15,000	5 ksi	20 ksi		5 ksi	7.5 ksi
Brittle Materials:							
Concrete	150	3,000	100 psi	1,350 psi			100 psi
Stone Masonry	165	1,000	10 psi	100 psi			10 psi
Brick Masonry	120	1,500	20 psi	300 psi			30 psi
Wood:							
Doug-Fir Larch North*							
• Joist & Rafters (No. 2)	35	1,700	650 psi	1,050 psi	625 psi	1,450 psi	95 psi
• Beams & Posts (No. 1)	35	1,600	700 psi	1,000 psi	625 psi	1,300 psi	85 psi
Southern Pine*							
• Joists & Rafters (No. 2)	35	1,600	625 psi	1,000 psi	565 psi	1,400 psi	90 psi
• Beams & Posts (D-No. 1)	35	1,600	1,050 psi	975 psi	440 psi	1,550 psi	110 psi
Hem-Fir*							
• Joists & Rafters (No. 2)	30	1,400	800 psi	1,050 psi	405 psi	1,150 psi	75 psi
• Beams & Posts (No. 1)	30	1,300	600 psi	850 psi	405 psi	1,000 psi	70 psi
Wood Products:							
Glu-Lam Beams	35	1,800	1,100 psi	1,650 psi	650 psi	2,400 psi	165 psi
Micro-Lam Beams	37	1,800	1,850 psi	2,460 psi	750 psi	2,600 psi	285 psi
Parallam Beams	45	2,000	2,000 psi	2,900 psi	650 psi	2,900 psi	290 psi

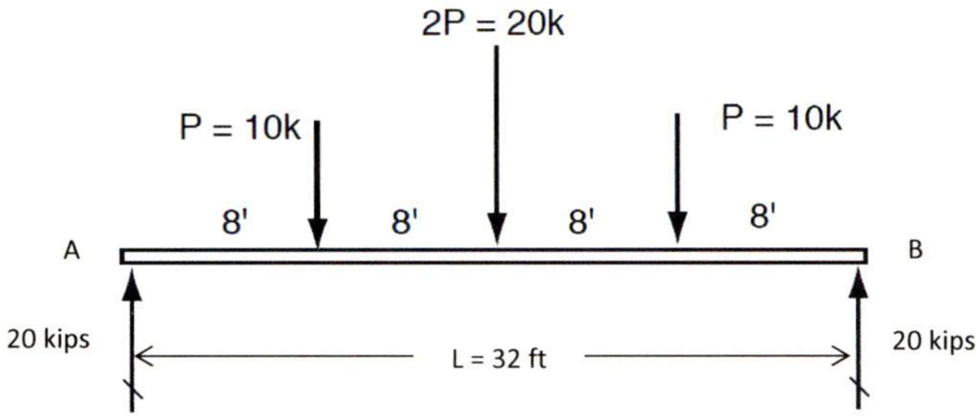
**Averaged stress values for design.*

Problem 4

Maximum Deflection Formulas (Use Superposition)

CONCENTRATED LOAD AT MIDSPAN		$\delta_{max} = \dfrac{PL^3}{48EI}$
		+
TWO CONCENTRATED LOADS SYMMETRICALLY SPACED		$\delta_{max} = \dfrac{Pa(3L^2 - 4a^2)}{24EI}$

4. A steel girder spanning 32 ft supports three concentrated beam reactions as shown. Design the lightest W14 x __ section for this girder. Check bending, shear and deflection. See the attached V & M diagrams.
$E = 30 \times 10^3$ ksi $\delta_{allow} = L/240$



Step 1.

Girder

A572, grade 50 steel.

$\sigma_y = 50$ ksi

$\sigma_{allow} = 30$ ksi

$\tau_{allow} = 20$ ksi

$\delta_{allow} = L/240 = 384\text{ in}/240 = 1.6\text{ in}$

$L = 32\text{ ft} \times \frac{12\text{ in}}{\text{ft}} = 384\text{ in}$

Step 2. V & M Diagrams

$V_{MAX} = 20$ kips

$M_{MAX} = 240\text{ kip}\cdot\text{ft} \times \frac{12\text{ in}}{\text{ft}} = 2880\text{ kip}\cdot\text{in}$

Step 3. $S_{req} = \frac{M_{MAX}}{\sigma_{allow}} = \frac{2880\text{ kip}\cdot\text{in}}{30\text{ ksi}} = 96\text{ in}^3$

Step 4. Table A-1(a)

W14×68 $S = 103\text{ in}^3$ (Lightest)

W14×74 $S = 112\text{ in}^3$

W14×82 $S = 123\text{ in}^3$

Select W14×68

Check Bending

$$\% = \frac{M_{wt}}{M_{MAX}} = \frac{\frac{wL^2}{8}}{M_{MAX}} = \frac{\frac{68\text{ lb/ft}\,(32\text{ ft})^2\left(\frac{\text{kip}}{1000\text{ lb}}\right)}{8}}{240\text{ kip ft}} = 0.0363 = 3.63\%$$

$$\frac{\text{Extra } S}{S_{req}} = \frac{103\text{ in}^3 - 96\text{ in}^3}{96\text{ in}^3} = 0.0729 = 7.3\% > 3.63\% \checkmark$$

OK, For Bending

Check Shear

W14×68

$d = 14.04\ in$

$t_w = 0.415\ in$

$$\tau_{avg} = \frac{V_{MAX}}{d t_w} = \frac{20\ kips}{14.04\ in (0.415\ in)} = 3.43\ ksi < \tau_{allow} = 20\ ksi$$

✓ ok, For Shear

Check Deflection

$$\delta_{allow} = \frac{384\ in}{240} = 1.6\ in$$

$E = 30\,000\ ksi$

$I = 723\ in^4$

By Superposition,

$$\delta_{MAX} = \frac{PL^3}{48EI} + \frac{Pa(3L^2 - 4a^2)}{24EI}$$

$$= \frac{20\ kip(384\ in)^3}{48(30\,000\ ksi)(723\ in^4)} + \frac{10\ kips(96\ in)\left[3(384\ in)^2 - 4(96\ in)^2\right]}{24(30\,000\ ksi)(723\ in^4)}$$

$$= 1.0877\ in + 0.7478\ in$$

$$= 1.84\ in > \delta_{allow} = 1.6\ in$$

Fail. Not Good

Try W14×74

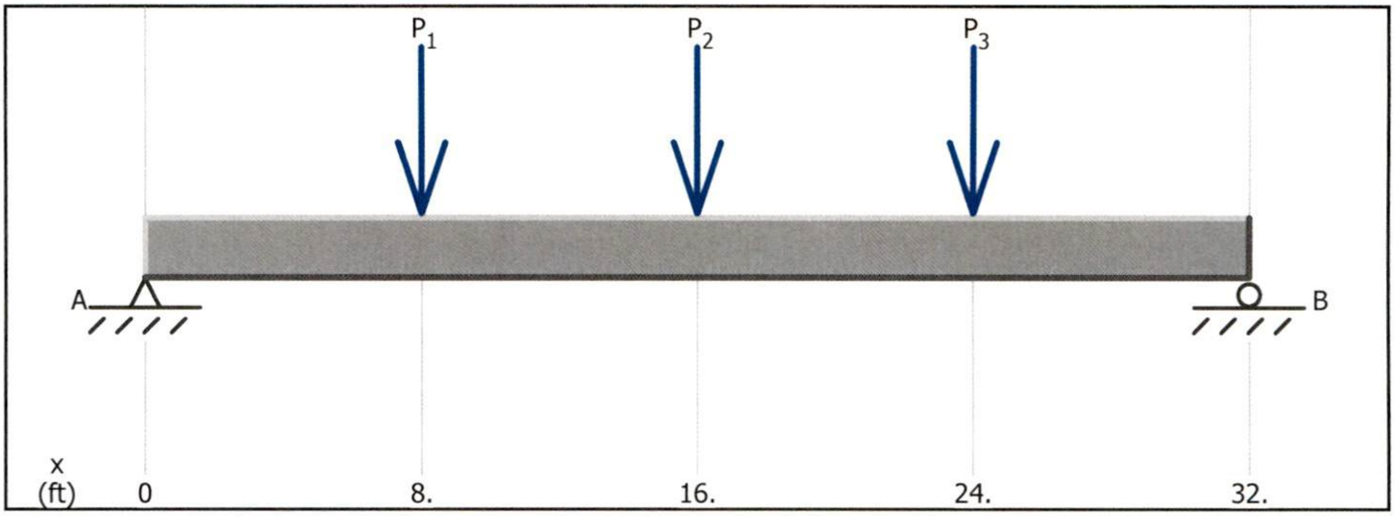
$$\delta_{MAX} = \frac{20\ kip(384\ in)^3}{48(30\,000\ ksi)(796\ in^4)} + \frac{10\ kips(96\ in)\left[3(384\ in)^2 - 4(96\ in)^2\right]}{24(30\,000\ ksi)(796\ in^4)}$$

$$= 0.98797 + 0.67924$$

$$= 1.67$$ close! Fails Deflec.

Use, W14×82

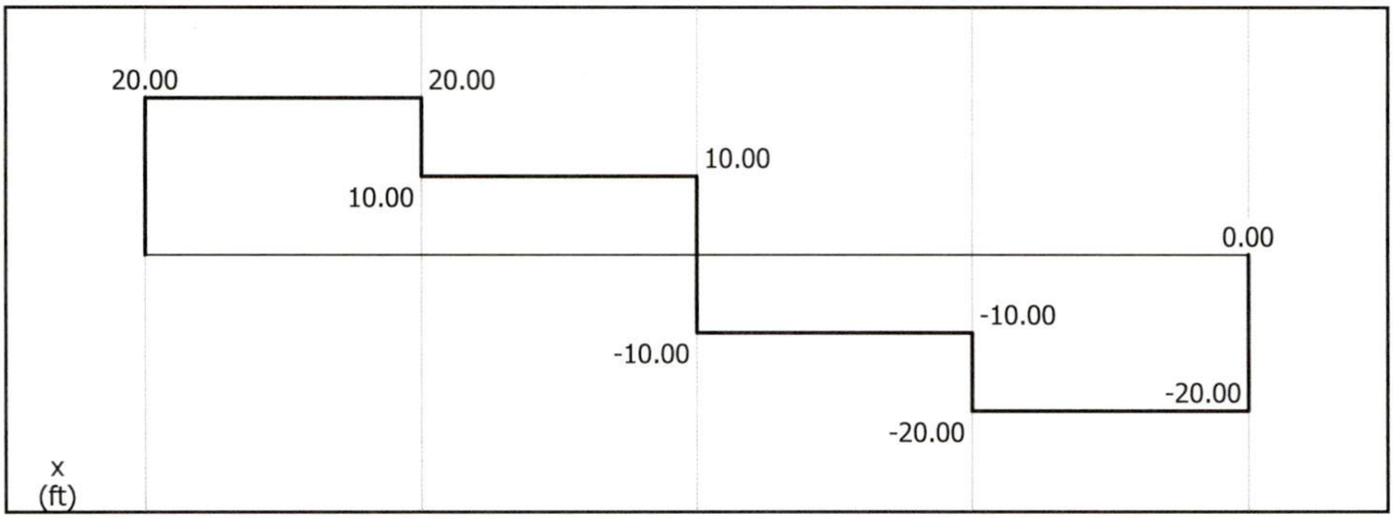
Problem 4 - V&M Diagrams



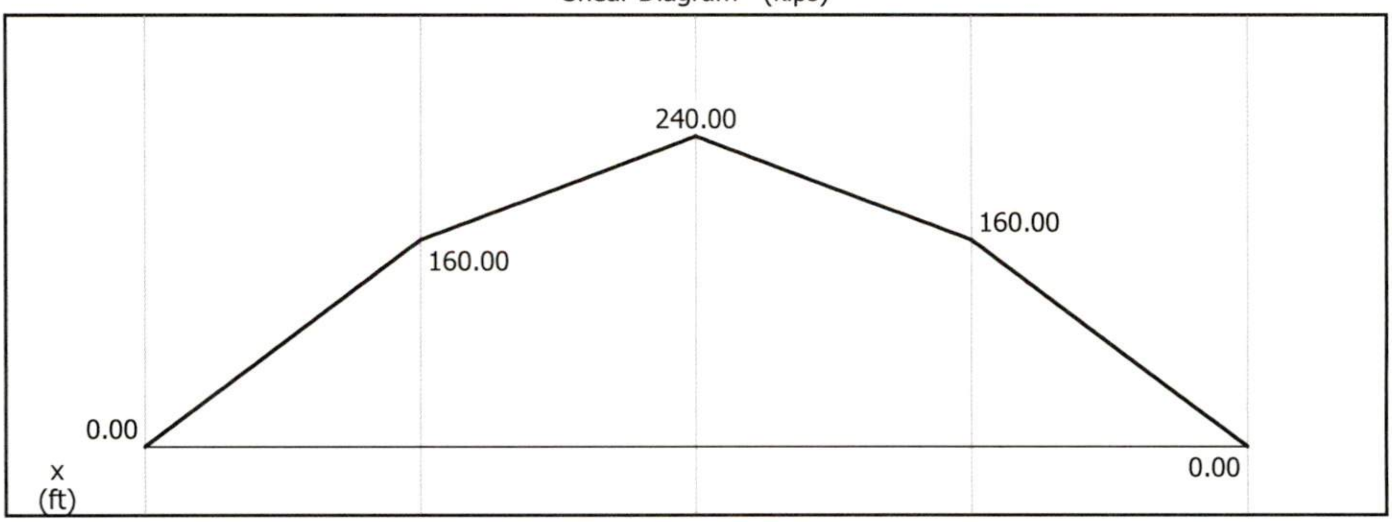
Load Diagram

P_1 = 10.0 kips (down)
P_2 = 20.0 kips (down)
P_3 = 10.0 kips (down)

A_y = 20.00 kips (up)
B_y = 20.00 kips (up)



Shear Diagram (kips)



Moment Diagram (kip-ft)